3. Word Puzzles and Games

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1. Word Games
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2. Computational Questions
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   - The Multiplication Principle
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3. References
As I was going to St. Ives,
I met a man with seven wives,
Each wife had seven sacks,
Each sack had seven cats,
Each cat had seven kits;
Kits, cats, sacks, and wives,
How many were going to St. Ives?
As I was going to St. Ives,
I met a man with seven wives,
Each wife had seven sacks,
Each sack had seven cats,
Each cat had seven kits;
Kits, cats, sacks, and wives,
How many were going to St. Ives?

Probably only one person: the “I” of the rhyme.
A Lipogram

Gadsby was walking back from a visit down in Branton Hill’s manufacturing district on a Saturday night. A busy day’s traffic had had its noisy run; and with not many folks in sight. His Honor got along without having to stop to grasp a hand, or talk; for a Mayor out of City Hall is a shining mark for any politician. And so, coming to Broadway, a booming bass drum and sounds of singing, told of a small Salvation Army unit carrying on amidst Broadway’s night shopping crowds. Gadsby, walking toward that group, saw a young girl, back toward him, just finishing a long, soulful oration . . .

Another Lipogram

Enfettered, these sentences repress free speech. The text deletes selected letters. We see the revered exegete reject metred verse: the sestet, the tercet — even \textit{les scènes élevées en grec}. He rebels. He sets new precedents. He lets cleverness exceed decent levels. He eschews the esteemed genres, the expected themes — even \textit{les belles lettres en vers}. He prefers the perverse French esthetes: Verne, Péret, Genet, Perec — hence, he pens fervent screeds, then enters the street, where he sells these letterpress newsletters, three cents per sheet. He engenders perfect newness wherever we need fresh terms.

Palindromes

- A man, a plan, a canal — Panama!
- Madam, I’m Adam.
- Was it a bat I saw?
- Rise to vote, sir.
- Draw, O Caesar, erase a coward.
- Egad, a base life defiles a bad age.
The oldest known anagrams may have been written in 260 BC by Lycophron, a Greek poet. His poem “Cassandra” contains two:

- The letters in the name of Ptolemy Philadelphus

\[\Pi\Omega\Lambda\E\M\A\I\O\Sigma,\]
\[\Lambda\Pi\O\ M\E\L\I\T\O\Sigma,\]

which translates to “made of honey.”

- The letters in the name of Arsinoë (Ptolemy’s queen),

\[\A\P\S\I\N\O\H,\]
\[\H\P\A\S\ I\O\N,\]

which means “Hera’s violet.”
Anagrams: American Presidents

- ABRAHAM LINCOLN: oh, call man “brain”
- HERBERT CLARK HOOVER: the ever dark horror
- HARRY S TRUMAN: rash army runt
- DWIGHT DAVID EISENHOWER: he did view the war doings
- JAMES EARL CARTER: a rare calm jester
- RONALD WILSON REAGAN: no darlings, no ERA law
- GEORGE HERBERT WALKER BUSH: huge berserk rebel warthog

More Anagrams

- INCOMPREHENSIBLE: problem in Chinese
- STORMY WEATHER: showery matter
- ROCKY MOUNTAINS: o, man — ski country
- THE PIANO BENCH: beneath Chopin

Anagrams

Unscramble these four Jumbles, one letter to each square, to form four ordinary words.

PUGOR
HAWSS
CLAICO
KUNFLY

Print answer here: HER

Yesterday’s Jumbles: WAGER GRIME INFANT GUITAR
Answer: When the storm hit, the church bells in the small town were — “RINGING” WET

Now arrange the circled letters to form the surprise answer, as suggested by the above cartoon.

What the Aging Beauty was able to keep when she had a face-lift.
Anagrams: Four Questions

1. How do you unscramble a jumbled word, like PUGOR?
2. Can we create an algorithm (or procedure) so that anyone, even a computer can unscramble a jumbled word?
3. How many anagrams of PUGOR are there? That is, in how many different ways can these five letters be rearranged as a single word?
4. How many anagrams does an $n$-letter word have? (Do all $n$-letter words have the same number of anagrams?)
The Multiplication Principle

In general, if a construction (or process) can be represented as a sequence of \( k \) steps, and the number of ways of performing each step is independent of any previous choices, and if \( n_1, n_2, n_3, \ldots, n_k \) denote the number of ways of performing each step, then

\[
N = n_1 \times n_2 \times n_3 \times \cdots \times n_k,
\]

denotes the total number of different constructions (or processes).

This statement is called \textit{the multiplication principle}.

The number of permutations of “abcd” is found by setting \( k = 4 \), with \( n_1 = 4 \), \( n_2 = 3 \), \( n_3 = 2 \), and \( n_1 = 1 \), hence

\[
N = n_1 \times n_2 \times n_3 \times n_4 = 4 \times 3 \times 2 \times 1 = 4!
\]
Some Problem Solving Heuristics

1. If the problem is too hard, try to solve an easier problem first.
2. *Divide and Conquer*: Hard problems can often be broken up into pieces, where each piece corresponds to an easier problem.
3. *Generalize*: Sometimes it is easier to solve a more general problem first.
4. *Make a picture*: Try to represent the problem as a picture.
5. *Construct an abstraction*. An abstraction is a simpler representation of the problem we are trying to solve. Reducing the problem to an abstraction that one has seen before often will suggest a simple solution.
A set is an unordered collection of objects (e.g., things, symbols, concepts, labels) in which each element (or member) appears no more than once. Matching left and right curly brackets, \( \{ \ldots \} \), are used to denote a set. Thus the set of odd numbers between 0 and 10 is \( \{ 1, 3, 5, 7, 9 \} \), and the set of suits in a standard deck of playing cards is \( \{ \spadesuit, \heartsuit, \clubsuit, \diamondsuit \} \). Since sets are unordered collections, \( \{ 1, 3, 5, 7, 9 \} \) and \( \{ 3, 1, 9, 5, 7 \} \) describe the same set.

The empty set is a special set that contains no objects at all. It is usually written as \( \emptyset \), or as \( \{ \} \).

A multiset is an unordered collection of objects in which each element can appear an arbitrary number of times. Thus \( \{ 1, 3, 3, 9 \} \) is a multiset, but not a set. (Is every set also a multiset?)

A sequence is an ordered collection of objects. Often we will use left and right matching parentheses to denote a sequence, e.g. the alphabet is usually represented by the ordered sequence \( (A, B, C, \ldots, Z) \).
Factorial Tree
The black dots are called *nodes* or *vertices*. The node at the top is called the *root*. The bottom nodes are called *leaves*. The colored lines are called *branches*, *edges*, or *links*. 
There are 6 anagrams using the letters $a$, $b$, $c$, one for each leaf node. Two begin with $a$, two begin with $b$, and two begin with $c$. 
There are 24 anagrams using the letters $a$, $b$, $c$, $d$ one for each leaf node. Six begin with $a$, six with $b$, six with $c$, and six with $d$. 
A Riddle (Revised)

As I was going to St. Ives,
I met a man with seven wives,
Each wife had seven sacks,
Each sack had seven cats,
Each cat had seven kits;
Kits, cats, sacks, and wives,
How many were leaving St. Ives?
Factorial tree with 5 different symbols

Number of leaf nodes: \( 5! = 5 \times 4 \times 3 \times 2 \times 1 = 5 \times 4! = 120. \)
Permutations of $n$ different symbols

Each time we add a symbol we create a new level in the tree.
Number of permutations of $n$ objects is

$$n! = n \times (n - 1)! = n \times (n - 1) \times (n - 2) \times \cdots \times 3 \times 2 \times 1.$$

This can be understood from either the perspective of factorial trees, or the previously defined multiplication principle.
Anagrams of words with repeated symbols
Anagrams of words with repeated symbols

\{a_1, a_2, b_1\}

\begin{align*}
  a_1 a_2 b_1 \\
  a_1 b_1 a_2 \\
  a_2 a_1 b_1 \\
  a_2 b_1 a_1 \\
  b_1 a_1 a_2 \\
  b_1 a_2 a_1
\end{align*}
Anagrams with repeated letters

How many anagrams of \textit{NEEDED} exactly match \textit{NEEDED}?

<table>
<thead>
<tr>
<th></th>
<th>EEE</th>
<th>EEE</th>
<th>EEE</th>
<th>EEE</th>
<th>EEE</th>
<th>EEE</th>
</tr>
</thead>
<tbody>
<tr>
<td>DD</td>
<td>NEEDED</td>
<td>NEEDED</td>
<td>NEEDED</td>
<td>NEEDED</td>
<td>NEEDED</td>
<td>NEEDED</td>
</tr>
<tr>
<td>DD</td>
<td>NEEDED</td>
<td>NEEDED</td>
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<td>NEEDED</td>
<td>NEEDED</td>
<td>NEEDED</td>
</tr>
</tbody>
</table>

By the multiplication principle, the number of times each anagram of \textit{NEEDED} is repeated equals $3! \cdot 2!$.

Consequently, the number of \textit{different} (or \textit{unique}) anagrams of \textit{NEEDED} equals

\[
\frac{6!}{3! \cdot 2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3! \cdot 2!} = 60.
\]
Anagrams with words with repeated symbols

- How many different anagrams are there of “abc”?
- How many different anagrams are there of “aaa”?
- How many different anagrams are there of “aab”?
- How many different anagrams are there of “abcd”?
- How many different anagrams are there of “aaaa”?
- How many different anagrams are there of “aaab”?
- How many different anagrams are there of “aabb”? 
Anagrams of words with repeated letters

Consider the word **MISSISSIPPI**. How many unique permutations of these letters exist? First we count the frequency (number of occurrences) of each letter:

<table>
<thead>
<tr>
<th>Letter</th>
<th>I</th>
<th>M</th>
<th>P</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

As a check, note that \( n = 4 + 1 + 2 + 4 = 11 \), the length of the word. The number of unique permutations is then,

\[
\frac{11!}{4! \cdot 1! \cdot 2! \cdot 4!} = 34,650.
\]

More generally, for a word that consists of \( k \) different letters of the alphabet, with \( r_1 \) repetitions of the first letter, \( r_2 \) of the second, \ldots, \( r_k \) of the \( k \)-th letter; the number of unique permutations equals

\[
\frac{(r_1 + r_2 + \cdots + r_k)!}{r_1! \cdot r_2! \cdots r_k!}.
\]
References


