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Mancala Games

In the heat of the day, the men will come forth, and sit themselves in companies, under the shady trees, to receive the fresh aire, and there passe the time in communication, having only one kind of game to recreate themselves withall, and that is in a piece of wood, certaine great holes cut, which they set upon the ground betwixt two of them, and with a number of some thirtie pibble stones, after a manner of counting, they take one from the other untill one is possessed of all, whereat some of them are wondrous nimble . . .

Mancala Games

There are countless varieties of mancala. It likely originated in Africa or the Middle East, and from there spread to the West Indies, to the Far East, to Indonesia and the Philippines. The game may date as far back as the stone age.

The game can be played in the dirt, or in boards carved out of wood or stone.

We will first consider the game known as wari, or oware, which is played on a board that has two rows of six pits. The following rules describe how the game is usually played in Barbados.

There are two players, here blue and red. Each player controls six pits, numbered 1 through 6, according to color, and a Store.
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Initially each numbered pit contains four seeds.
There are two players, here blue and red. Each player controls six pits, numbered 1 through 6, according to color, and a Store.

Initially each numbered pit contains four seeds.

Players alternate turns.
Initially each numbered pit contains four seeds.
Players alternate turns.
During each turn a player selects one of the numbered pits that contains some seed, and then sews the seeds, placing one per pit but skipping over the stores, in a counter-clockwise manner.
During each turn a player selects one of the numbered pits that contains some seed, and then sews the seeds, placing one per pit but skipping over the stores, in a counter-clockwise manner.

If the last seed lands in a pit in the opponent’s row, and that pit contains 2 or 3 seeds, including the one just placed therein, then the player captures these seeds by moving them into his or her store.
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If in addition one or more pits immediately clockwise to this one lie in the same row, and contain two or three seeds, then their contents are also transferred to the store.
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A player is not allowed to select a move that would prevent his or her opponent from moving.
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Play continues until, despite the previous rule, one player is unable to move. In this case all the remaining tokens, which lie on one player’s side of the board, are transferred into the store that belongs to that player.
Wari: Rules

Play continues until, despite the previous rule, one player is unable to move. In this case all the remaining tokens, which lie on one player’s side of the board, are transferred into the store that belongs to that player.

The player with the largest store of seeds wins.
Alice selects Pit 4 (shown by the red arrow).
Wari: Example Game: Alice vs. Bob

Alice selects Pit 4 (shown by the red arrow). Alice then grabs all four seeds in the indicated pit, and sows one into each of the four following pits, in a counter-clockwise manner, skipping over the store.
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Wari: Example Game: Alice vs. Bob

Alice selects Pit 4 (shown by the red arrow). Alice then grabs all four seeds in the indicated pit, and sows one into each of the four following pits, in a counter-clockwise manner, skipping over the store. The last seed falls into Pit 2, which now contains 5 seeds. Since the number of seeds is not 2 or 3, her turn is over.
It’s now Bob’s turn. He selects Pit 6.
It’s now Bob’s turn. He selects Pit 6. Bob now removes all the seeds contained in Pit 6, and sows them over the board, in a counter-clockwise manner, placing one seed in each pit, skipping over the store, until his hand is empty.
Wari: Example Game: Alice vs. Bob

It’s now Bob’s turn. He selects Pit 6. Bob now removes all the seeds contained in Pit 6, and sows them over the board, in a counter-clockwise manner, placing one seed in each pit, skipping over the store, until his hand is empty.
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### Wari: Example Game: Alice vs. Bob

<table>
<thead>
<tr>
<th>Store</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
</table>

Alice now selects Pit 3.

As the pit in which the last seed fell now contains six seeds, Alice's turn is over.


14. Mancala
Wari: Example Game: Alice vs. Bob

Store  6  5  4  3  2  1

Alice now selects Pit 3. Alice then removes all of five seeds from this pit, and sows them, again in a counter-clockwise manner, placing one seed into every numbered pit (ignoring the stores), until her hand is empty.
Alice now selects Pit 3. Alice then removes all of five seeds from this pit, and sows them, again in a counter-clockwise manner, placing one seed into every numbered pit (ignoring the stores), until her hand is empty.
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Wari: Example Game: Alice vs. Bob

Bob selects pit 1 which now contains six seeds.
Bob selects pit 1 which now contains six seeds. Bob now sows the seeds in the usual manner.
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Bob selects pit 1 which now contains six seeds. Bob now sows the seeds in the usual manner. The last seed from Bob’s Pit 1 falls into Alice’s Pit 1, which now contains six seeds. Thus Bob’s turn is over.
Seeing an opportunity, Alice selects Pit 6, which contains six seeds.
Seeing an opportunity, Alice selects Pit 6, which contains six seeds. She sows one seed into each pit.
**Wari: Example Game: Alice vs. Bob**

<table>
<thead>
<tr>
<th>Store</th>
<th>6</th>
<th>5</th>
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Seeing an opportunity, Alice selects Pit 6, which contains six seeds. She sows one seed into each pit.
Wari: Example Game: Alice vs. Bob

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Wari: Example Game: Alice vs. Bob

Seeing an opportunity, Alice selects Pit 6, which contains six seeds. She sows one seed into each pit. Note that the last pit contains two stones. Thus she captures these and
Seeing an opportunity, Alice selects Pit 6, which contains six seeds. She sows one seed into each pit. Note that the last pit contains two stones. Thus she captures these and places them in her store. After the capture, the turn passes to Bob.
Seeking revenge, Bob selects Pit 4.
Wari: Example Game: Alice vs. Bob

Seeking revenge, Bob selects Pit 4. He drops seeds, one by one, counter-clockwise, into each successive pit.
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Seeking revenge, Bob selects Pit 4. He drops seeds, one by one, counter-clockwise, into each successive pit. The last seed falls into Alice’s Pit 4. Since the total number of seeds in Pit 4 equals three, Bob captures these,
Seeking revenge, Bob selects Pit 4. He drops seeds, one by one, counter-clockwise, into each successive pit. The last seed falls into Alice’s Pit 4. Since the total number of seeds in Pit 4 equals three, Bob captures these, and places them in his store.
Alice now selects Pit 1.
Alice now selects Pit 1. Sowing these six seeds, one per pit, in a counter-clockwise direction,
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Wari: Example Game: Alice vs. Bob

Alice now selects Pit 1. Sowing these six seeds, one per pit, in a counter-clockwise direction,
Alice now selects Pit 1. Sowing these six seeds, one per pit, in a counter-clockwise direction, she drops the last seed into Bob’s Pit 2. Since the last pit contains nine seeds, her turn is over.
Bob selects his Pit 1, which only contains two seeds.
Wari: Example Game: Alice vs. Bob

Bob selects his Pit 1, which only contains two seeds. Consequently, his turn is quickly over.
Bob selects his Pit 1, which only contains two seeds. Consequently, his turn is quickly over.
Alice carelessly selects her Pit 6, which contains just a single seed.
Alice carelessly selects her Pit 6, which contains just a single seed. She advances it forward to Bob’s side of the board.
Bob now strikes by selecting his Pit 3, which contains seven seeds.
Bob now strikes by selecting his Pit 3, which contains seven seeds. Sewing them around the board, find the last seed in Alice’s Pit 4.
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Bob now strikes by selecting his Pit 3, which contains *seven* seeds. Sewing them around the board, find the last seed in Alice’s Pit 4.
Bob now strikes by selecting his Pit 3, which contains seven seeds. Sewing them around the board, find the last seed in Alice’s Pit 4.
Bob now strikes by selecting his Pit 3, which contains seven seeds. Sewing them around the board, find the last seed in Alice’s Pit 4. As Pit 4 has two seeds, and the one before it has three, Bob captures all five.
Bob now strikes by selecting his Pit 3, which contains *seven* seeds. Sewing them around the board, find the last seed in Alice’s Pit 4. As Pit 4 has *two* seeds, and the one before it has *three*, Bob captures all *five*. After placing them in his store, his turn is over.
Wari: Additional Rules

- Seeds can only be captured when the last seed sown lands in the opponent’s row, in a pit that contains 2 or 3 seeds (counting the seed just added).
- If a player selects a pit with more than eleven seeds, then the original pit must be skipped as the seeds are sown. Thus, the selected pit should always be empty at the end of each turn.
- Whenever possible, a player must always move so that at least one seed remains on the opposite side of the board. For example, a player may not eliminate every seed from the opponent’s row by a multiple capture. (In this case, only the seeds in the last pit are captured; or none at all, if this is the only occupied pit, and other seeds remain on the board.)
- If a player cannot move despite the previous rule, then the game ends, and the he or she captures all of the remaining seeds.
- The player with the greatest number of seeds wins.
Mancala: Questions

How hard would it be to analyze this game?

How many states (i.e., positions) are there?

How large is the game tree?
How many states are in wari?

Let $p$ denote the number of pits, and $s$, the number of seeds. (In our version of wari, $p = 2 \times 6 = 12$ and $s = p \times 4 = 48$.) We will represent pits by using dividers, each indicated by a **brown** vertical bar: . Note that one divider separates two pits,

![Divider between Pit 1 and Pit 2](image)

two dividers separate three pits,

![Dividers between Pit 1, Pit 2, and Pit 3](image)

and so on. Thus $p - 1$ bars are required to separate $p$ pits.

We will represent seeds by **green** balls: . Thus one way of distributing 8 seeds over 6 pits is represented by,

![Seeds in Pit 1, 2, 3](image)

(Here Pits 1, 6, 7, and 8 are empty, Pits 2 and 5 each contain one seed, Pit 3 contains two seeds, and Pit 4 contains four seeds.)
How many states are in wari?

In fact there is a one-to-one correspondence between the distributions of $s$ indistinguishable seeds over $p$ pits, and the anagrams of $s + p - 1$ letter words consisting of $s$ symbols, and $p - 1$ symbols. From our anagram formula, the number of ways of distributing $s$ seeds over $p$ pits is thus,

$$\binom{s + p - 1}{s} = \binom{s + p - 1}{p - 1} = \frac{(s + p - 1)!}{s!(p - 1)!}.$$ 

With $s = 48$ and $p = 14$ (including the two stores), we see that there are

$$\frac{(48 + 14 - 1)!}{48! \cdot (14 - 1)!} = \frac{61!}{48! \cdot 13!} = 6,566,222,272,575.$$ 

states.
The game of bao is a variant of mancala that is played on a board that contains four rows of eight pits. It is very popular in East Africa. Although the geographic distribution of bao is more limited than warri, there still exist a variety of rule sets. We shall describe a version that is played in Zanzibar (Voogt, 2005).

Bao

The board consists of four rows of eight pits. Note that two of the pits are square shaped. Each square pit is called a *nyumba*. In the initial configuration, each *nyumba* contains 6 seeds (or *kete*), and the two adjacent pits in each row contain 2 *kete*, as shown. Each player holds 22 additional *kete*, in a reserve store.
Bao

During the game, each player only sows seeds in the two rows on that player's side of the board. Thus the \textit{North} player sows seeds in her \textit{front} row \textit{A}, and \textit{back} row \textit{B}. Likewise, the other \textit{South} player sows seeds in his \textit{front} row \textit{a}, and \textit{back} row \textit{b}. Columns are indexed 1 through 8, from left to right, as shown.

\begin{center}
\begin{tabular}{cccccccc}
\hline
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
\textit{North} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 2 & 2 & 6 & 0 & 0 & 0 & 0 \\
\textit{a} & 0 & 0 & 0 & 0 & 6 & 2 & 2 & 0 \\
\textit{b} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
\end{tabular}
\end{center}

Initial configuration: each player has 10 \textit{kete} in their front row, as shown.

Note that \textit{North}'s \textit{nyumba} is \textit{A5}, while \textit{South}'s is \textit{a5}.

The outermost front pits, \textit{A1}, \textit{A8}, \textit{a1}, and \textit{a8}, are called the \textit{kichwa}; the next outermost front pits, \textit{A2}, \textit{A7}, \textit{a2}, and \textit{a7}, are called the \textit{kimbi}.

The goal of the game is to remove every seed from your opponent's front row.
Bao: Sample Game

Bao is much more complicated than wari. Thus, we will first illustrate a game, and describe the rules as players *North* and *South* play a game.¹

Each game has two phases. In the *namua*, a turn consists of a player depositing one *kete* (from his store) into one of the pits on his or her front row that already contains one or more seeds.

*South* begins the game turn by dropping a seed from his store into pit *a6*.

```
<table>
<thead>
<tr>
<th>8</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
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<tbody>
<tr>
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<td>2</td>
<td>6</td>
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</tbody>
</table>
```

*South's* first move: *a6* (in progress).

¹This game actually appears in (Vooght, 2005) as a Master Tournament Bao game that was played in Zanzibar in 1994.
Bao: Sample Game

If *North* had seeds in the pit opposite *a6*, i.e. *A3*, then *South* would now capture them. Since *A3* is empty, *South* continues his first turn by removing all three seeds from *a6*, and sowing them, one by one, into pits *a5*, *a4*, and *a3*, just like in *wari*. (In bao, however, player’s have the option of sowing seeds in either the clockwise, or counterclockwise directions.)

```
8 7 6 5 4 3 2 1
0 0 0 0 0 0 0 0
0 2 2 6 0 0 0 0
a 0 0 1 1 7 0 2 0
b 0 0 0 0 0 0 0 0
```

*South*’s first move, continued.

Since the last seed fell into an empty pit (*a3*), *South*’s turn is over.
**Bao: Sample Game**

As in checkers, players must capture an opponent's seeds whenever possible. Thus *North* must place a seed in either *A5* or *A6*. Here, *North* plays *A5*, defending her *nyumba*.

<table>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
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</tr>
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<td>1</td>
<td>7</td>
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<td><strong>b</strong></td>
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</tr>
</tbody>
</table>

*North* thus captures the opposite seed in pit *a4*. Since this seed is taken from one of the center four columns, *North* may opt to either sow the first captured seed into either *kichwa* (*A1* or *A8*). In this case, *North* elects *A8*. Since the last (and only) seed lands in an empty pit, *North*'s first turn is over.

<table>
<thead>
<tr>
<th></th>
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<tr>
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<td>7</td>
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<td>7</td>
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<td><strong>b</strong></td>
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</tr>
</tbody>
</table>

At the beginning of his next turn, *South* must place a seed from his store into *a3*, to capture the two seeds in *A6*. 
Bao: Sample Game

Because these are taken from one of the four center columns, he can choose to sow these seeds into his front row, beginning from either *kichwa*. He chooses to sow the captured seeds from right to left, and thus places one seed in \(a8\) and the second in \(a7\). (Captured seeds must be sown across the front row.)

<table>
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<td>0</td>
<td>7</td>
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<td>3</td>
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<tr>
<td>B</td>
<td>0</td>
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</tr>
</tbody>
</table>

Since the last seed was placed into an occupied pit \((a7)\), *South*’s turn continues. If there were seeds in the opposite pit \((A2)\), then *South* would capture them too. (This would be called a *multiple capture*.) However, in this case, \(A2\) is empty. So *South* removes all of the seeds from the last pit, \(a7\), and sows them, one by one, into the subsequent pits \((a6, a5, \text{and} a4)\), in the same direction as before.

<table>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>8</td>
<td>1</td>
<td>0</td>
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<td>B</td>
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</tr>
</tbody>
</table>
North's next move is likewise forced to be A5. Why?

After placing a seed in A5 from her stock, she captures the singleton from a4. The latter could be deposited in either kichwa. She chooses her left side: A1. Since this pit was previously empty, her turn is over.

As South considers his next move, what are his options?
Because player’s are obliged to capture, *South’s* next move must be *a8*, capturing the single opposite seed in pit *A1*. Seeds captured from a *kichwa* or a *kimbi*, must be sown across the player’s front row, starting from the nearest *kichwa*. *South* drops his prize in *a8*. 
Bao: Sample Game

Since \textit{a8} contained three seeds, \textit{South} sows them in \textit{a7}, \textit{a6}, and \textit{a5}, his \textit{nyumba}.

\begin{center}
\begin{tabular}{cccccccc}
\hline
 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\
\hline
\textit{B} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\textit{A} & 1 & 2 & 0 & 8 & 0 & 0 & 0 & 0 \\
\textit{a} & 0 & 0 & 2 & 0 & 9 & 2 & 1 & 0 \\
\textit{b} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
\end{tabular}
\end{center}

In this interesting situation, \textit{South}'s last seed was dropped into \textit{a5}, the \textit{nyumba}. If this were an ordinary pit, then \textit{South} would be obliged to remove all of the seeds from the last pit, and resow them in the original direction. Special rules, however, apply to the \textit{nyumba}. In this case, a player may choose not to resow the \textit{nyumba} and thus end his turn. Thus, \textit{South} elects to end his turn. (Note: once your \textit{nyumba} has been emptied once, this rule no longer applies.)

Now, \textit{North} cannot capture. A situation in which you cannot capture (like the first move in this game) is called a \textit{takasa}. During a \textit{takasa}, you must place a seed in a pit that contains one or more of your seeds. (If possible, this pit should not be the \textit{nyumba}.) The player then removes all of the seeds from this pit, and may sow them in either the clockwise or counterclockwise direction, starting from the next pit in that direction. If the last seed falls into an occupied pit, then the process continues in the original direction. It is not possible to capture seeds from your opponent during a \textit{takasa}. 

Thus, *North* elects to drop a seed from her stock into \( A_7 \), and resow the three seeds therein to her left, clockwise, into \( A_6, A_5, \) and \( A_4 \). Since the last pit, \( A_4 \), was empty, her turn is over.

What are *South’s* possible moves? Which move is best?
Indeed, *South* should now protect his *nyumba* by playing *a5*, and capturing the seed in *A4*. He chooses to resow the captured seed in his left *kichwa*. Since pit *a1* was empty, his turn is over; he does not capture the seed in *A8*.

What are *North*’s possible moves?
North can capture seeds by playing either $A_6$ or $A_8$. Here she opts for the latter. (Why?) The captured seed must be placed in her nearest *kichwa*, $A_8$. Since the *kichwa* was occupied, the three seeds contained therein must be resown across the front row.

Note that the last seed was dropped into her *nyumba*, which she elects to empty. The seeds must be sown in the same direction as before (counterclockwise), continuing into her back row.
Bao: Sample Game

Since \textit{North}'s last seed falls into an empty pit, \textit{B6}, her turn is over.

\begin{center}
\begin{tabular}{cccccccc}
8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\
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0 & 1 & 2 & 0 & 1 & 1 & 1 & 1 \\
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\textit{a} & & & & & & & \\
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\textit{b} & & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\end{tabular}
\end{center}

\textit{South} now plays \textit{a3}, and selects his right \textit{kichwa}, \textit{a8}. Thus the two captured seeds from \textit{A6} are sown in pits \textit{a8} and \textit{a7}. Since the latter pit was occupied, \textit{South} now captures the seed in \textit{A2}. This seed is then dropped into the \textit{a8}, likewise capturing \textit{A1}. This seed is subsequently dropped into \textit{a8}. Since \textit{A1} is empty, \textit{South} must redistribute the three seeds in \textit{a8} to \textit{a7}, \textit{a6}, and finally, \textit{a5}, capturing \textit{A4}. Although this captured seed is from the center of the board, \textit{South} must resow it the original direction. (The only exception to this would occur if a left \textit{kimbi} or \textit{kichwa} was captured.) Thus, \textit{South} brings the captured seed back into \textit{a8}, which is empty, and so ends his turn. After the dust has settled,
North must now drop a seed from her stock into pit A3, capturing three seeds from a6. She elects to resow these from her right kichwa, into A8, A7, and A6, ending her turn.

What move must South execute?
South must now drop a seed from his stock into pit a3, capturing one seed from A6. He elects to deposit this seed into his left, unoccupied kichwa, ending his turn.

What move must North execute?
### Bao: Sample Game

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North must now drop a seed from her stock into pit A8, capturing one seed from a1. She must deposit this seed into her right kichwa, A8, which now contains three seeds. North must now resow these into pits A7, A6, and A5. Since the latter was empty, her turn is over.

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What move must South execute?
**Bao: Sample Game**

South must now drop a seed from his stock into pit $a3$, capturing one seed from $A6$. He chooses to deposit this seed into his right *kichwa*, $a8$, which now already contains one seed. Since the opposite pit $A1$ is empty, he resows these two seeds into $a7$ and $a6$. Since the latter was empty, his turn is over.

What move must *North* execute?
North must now drop a seed from his stock into pit A3, capturing one seed from a6. She chooses to deposit this seed into her left kichwa, A1. Since the latter was empty, her turn ends.

What moves can South execute?
South can deposit a seed into either a3 or a7. Since either move is a takasa, he must resow the chosen pit, in the direction of his choice. In this case, he deposits a seed into a7 from his stock, and resows these five seeds to the right (clockwise), into his back row.

What moves can North execute?
North must capture the seed in a8 by depositing a seed in her kichwa at A1. The captured seed is dropped in A1. Since the latter was occupied, the three seeds therein are resown into A2, A3, and A4. Since the latter was empty, her turn is over.

What should South do?
South must now capture the seed in **A4** by depositing a seed in his *nyumba*, **a5**. He can choose to return the captured seed to either *kichwa*. (Why?) He selects the right one, **a8**. Since the latter was empty, his turn is over.

What should **North** do?
Since *North* cannot capture any seeds, she must execute a *takasa*, by placing a seed from her stock into an occupied pit on her front row: either *A2*, *A3*, *A5*, or *A7*. Note that because she emptied her *nyumba* in an earlier move, she now treats *A5* as an ordinary pit: the special rules of the *nyumba* no longer apply to her.

Here, *North* decides to add a seed to *A3*, and then to resow the five seeds there in a counterclockwise direction, into *A4*, *A5*, *A6*, *A7*, and *A8*. Since the latter was empty, her turn ends.
Bao: Sample Game

South has two capturing moves, \( a_3 \) and \( a_5 \). Since it is advisable to always protect the most populated pit, he elects to place a seed from his store into pit \( a_5 \). He then places the captured seed (from \( A_4 \)) into his left kichwa, \( a_1 \). Since the latter was empty, his turn is over.

What should North do?
**Bao: Sample Game**

North has two capturing moves as well, $A6$ and $A8$. Since the prize for the former is greater, North places a seed from her store into pit $A6$ and captures the five seeds in $a3$. She then chooses to bring them in using her right *kichwa*, by placing seeds in $A8$, $A7$, $A6$, $A5$, and finally $A4$. Since the last pit was empty, her turn is over.

How should *South* respond?
South again has two capturing moves, \textit{a1} and \textit{a5}. Again, he protects his \textit{nyumba} by placing a seed from his store in \textit{a5}. He then places the captured seed (from \textit{A4}) into his right \textit{kichwa}, \textit{a8}. Since the latter is occupied (but \textit{A1} is empty), he resows these two seeds into pits \textit{a7} and \textit{a6}. As the latter was previously empty, his turn expires.

What can \textit{North} do?
Bao: Sample Game

Likewise, North has two capturing moves as well, A2 and A8. This time North places a seed from her store into her left kimbi, A2, and captures the singleton in a7. The captured seed must be dropped in her left kichwa, A1. Since the latter was previously empty, her turn ends.

How should South respond?
Bao: Sample Game

South is obliged to perform the only possible capture. He thus places a seed from his store into \( a1 \), and captures two seeds from \( A8 \). These seeds must be resown starting from his left \textit{kichwa}, into \( a1 \), and the empty \( a2 \), ending his turn.

What must \textit{North} do?
Since North has only one capturing move, she must place a seed from her store in her right kimbi, A7, capturing the singleton from a2. The captured seed must then be placed in her rightmost kichwa, A8. Since the latter was previously empty, her turn is over.

How must South respond?
South is again obliged to perform the only possible capture. He thus places a seed from his store into $a_1$, and recaptures one seed from $A_8$. This seed must be resown into his left kichwa, into $a_1$. Since this pit was occupied, South must resow these five seeds into pits $a_2$, $a_3$, $a_4$, $a_5$, and $a_6$. Since one seed was already present in the last pit, and no seeds appear in the opposite one in $A_3$, South must resow these two seeds into pits $a_7$ and $a_8$. Since the latter pit is empty, South’s turn is over.

What moves can North perform?
North can now choose between five different capturing moves. She decides to deposit a seed from her store, in pit A2, capturing the seed from a7, which must then be placed in her left kichwa, A1. Since this pit was previously occupied, North's also captures the seed in a8. The latter is placed in her left kichwa, A1. Since the latter was occupied, and a8 is now empty, North resows these three seeds into pits A2, A3 and A4. Since the latter was empty, her turn is over.

How should South respond?
Bao: Sample Game

South has a choice between four different capturing moves. He chooses to protect his nyumba, by depositing a seed from his store into pit a5. He then chooses to drop the captured seed (from A4) into his left kichwa, a1, ending his turn.

What moves can North perform?
Bao: Sample Game

North now has a choice of three capturing moves: A5, A6, and A7. North chooses to deposit a seed from her store into pit A6, capturing one seed from a3. The latter she sows into her left kyumba, A1, ending her turn.
South has a choice between two capturing moves. He elects to deposit a seed from his store into $a2$, capturing the six seeds that lie in $A7$. These six seeds must be sown starting with the left kichwa into pits $a1$, $a2$, $a3$, $a4$, $a5$, and $a6$. Since the latter was empty, this ends his turn.
North can choose between three different capturing moves. She elects to drop a seed from her store into pit A3, capturing the seed in a6. By placing the captured seed into her left kichwa, she facilitates the following combination. First she must resow the two seeds from A1 into A2 and A3. Since the latter was occupied, the three seeds are removed from A3, and are resown into A4, A5, and A6. Since the latter was occupied, she captures the one seed in a6, which must be placed in her left kichwa, A1. Since the latter was empty, this ends her turn.
South again chooses to protect his *nyumba* by dropping a seed into pit *a5*, capturing the seed in *A4*. He elects to drop the captured seed into his right *kichwa* at *a8*, ending his turn.
North can now choose between three different capturing moves. She decides to deposit a seed from her store, into pit A5, capturing two seeds from a4. She chooses to sow the captured seed from her left kichwa into pits A1 and A2. Since the latter was occupied, and no seeds are located in the opposite pit a7, she resows these six seeds into pits A3, A4, A5, A6, A7, and A8, which ends her turn.
Bao: Sample Game

South again chooses to protect his nyumba by dropping a seed into pit $a_5$, capturing the seed in $A_4$. He elects to drop the captured seed into his left kichwa at $a_1$. In turn, he captures the seed in $A_8$, which is also dropped into pit $a_1$. Then these four seeds are resown into $a_2$, $a_3$, $a_4$, green $a_5$. Since the last seed landed in the nyumba, South could elect to end his turn. However, this time for once, he chooses to resow the 20 seeds contained here. In doing so, these seeds must be resown in the same (clockwise) direction, around and around; the last seed landing in $b_{vii}$.

South’s fun continues, as he now must resow the three seeds in pit $b_8$ into, $b_7$, $b_6$, and $b_5$. The latter pit, which now also contains three seeds must be resown into $b_4$, $b_3$, and $b_2$. Likewise, the latter pit, which now contains two seeds, must be resown into $b_1$ and $a_1$. Likewise, the two seeds in $a_1$ must be resown into $a_2$ and $a_3$. The latter move forces South to capture the six seeds in $A_6$. These are now sown from the left kichwa into pits $a_1$, $a_2$, $a_3$, $a_4$, $a_5$, and $a_6$. The last move results in the capture of the seed in $A_3$, which must be dropped in $a_1$. Since the latter is occupied, the two seeds therein are resown into pits $a_2$ and $a_3$. The latter six seeds are then sown into $a_4$, $a_5$, $a_6$, $a_7$, $a_8$, $b_8$, ending this turn.
**Bao: Sample Game**

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**North** can choose between three different capturing moves. She elects to drop a seed from her store into pit *A6*, capturing the four seeds in *a3*. She then sows the four captured seeds from her left *kichwa* into pits *A1*, *A2*, *A3*, and *A4*, ending her turn.

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South again chooses to protect his *nyumba* by dropping a seed into pit *a5*, capturing the seed in *A4*. He elects to drop the captured seed into his right *kichwa* at *a8*, ending his turn.
References


