1. (35 pts.) A motor turns a bar. A uniform bar of length $l$ and mass $m$ is turned by a motor whose shaft is attached to the end of the bar at $O$. The angle that the bar makes (measured counter-clockwise) from the positive $x$ axis is known as a function of time, $\theta(t) = 2\pi t/\omega_0^2$. Neglect gravity.

   a.) (10 pts.) Draw a free body diagram of the bar at $t = 1$ sec.

   b.) (5 pts.) Find $\vec{\omega}$, the angular velocity, and $\vec{\alpha}$, the angular acceleration, of the bar at $t = 1$ sec.

   c.) (5 pts.) Find the acceleration of the center of mass of the bar at $t = 1$ sec.

   d.) (5 pts.) Find $\vec{F}_O$, the force acting on the bar from the motor at point $O$, at $t = 1$ sec.

   e.) (5 pts.) Find $M_O$, the torque applied to the bar from the motor at point $O$, at $t = 1$ sec.

   f.) (5 pts.) Find the power into the system motor at $t = 1$ sec. (Hint: $P_{in} = \dot{E}_K$, where $P_{in} = \sum \dot{M}_O \cdot \vec{\omega}$ and $\dot{E}_K = (1/2)I^O \dot{\omega}^2$.)

2. (30 pts.) Massless pulley, dumbbell, and a hanging mass. A block of mass $m$ falls vertically but is supported by an inextensible massless string which is wrapped around an ideal (massless, frictionless) pulley with radius $a$ and center at $O$. The pulley is welded to a dumbbell. The dumbbell is made of a massless rod welded to uniform solid spheres at $A$ and $B$, each of radius $R$ and mass $M$ and each of whose center is a distance $\ell$ from $O$. At the instant in question, the dumbbell is known to make an angle $\theta$ with respect to the positive $x$ axis and to be spinning at rate $\dot{\theta}$. Point $C$ is a distance $h$ down from $O$. There is gravity.

   a.) (10 pts.) Draw separate free body diagrams of (i) the dumbbell plus pulley, and (ii) the block.

   b.) (10 pts.) Find $I^A_C$, the polar moment of inertia of the dumbbell with respect to point $O$ in terms of some or all of $m$, $M$, $a$, $R$, and $\ell$.

   c.) (10 pts.) Find the $\vec{a}_B$, the acceleration of the block of mass $m$, in terms of some or all of $m$, $M$, $a$, $R$, $\ell$, $g$, $\dot{\theta}$, $i$, and $j$.

3. (35 pts.) Circular rigid body motion: brake arm and rotating cylinder. A brake arm of negligible mass and the dimensions shown is pinned frictionlessly at point $Q$. At $t = 0$, when the brake arm contacts the top of the cylinder of radius $R$ and mass $m$ at point $C$, the cylinder is turning at rate $\omega_0$, CW. The cylinder is supported by a frictionless hinge at $O$. The coefficient of friction between the brake arm and the cylinder is $\mu$. A force $P$ is applied to the end of the brake arm as shown in a direction perpendicular to the bar throughout its motion. Neglect gravity.

   a.) (10 pts.) Draw separate FBD's of the brake arm and the cylinder.

   b.) (10 pts.) Find $\alpha$, the angular acceleration of the cylinder, in terms of some or all of $m$, $R$, $h$, $\ell$, $d$, $\mu$, $\omega_0$, and $P$.

   c.) (10 pts.) Find $t_f$, the time it takes for the cylinder to come to rest in terms of some or all of $m$, $R$, $h$, $\ell$, $d$, $\mu$, $\omega_0$, and $P$.

   d.) (5 pts.) Find $\vec{F}_O$, the reaction of the hinge on the cylinder at $O$ in terms of some or all of $m$, $R$, $h$, $\ell$, $d$, $\mu$, $\omega_0$, $P$, $i$, and $j$. 
1. a) \( \mathbf{t} = 1 \text{ sec} \Rightarrow \mathbf{a} = 2\pi \text{ rad} \).

b) \( \mathbf{\dot{w}} = \theta \mathbf{k} = \frac{4\pi}{\text{sec}^2} \mathbf{k} \) @ \( t = 1 \text{ sec} \)

\[ \mathbf{z} = \mathbf{\dot{w}} = \frac{4\pi}{\text{sec}^2} \mathbf{k} = \text{const}. \]

c) \( \mathbf{a}_{G} = \alpha \times \mathbf{r}_{O} - \omega^2 \mathbf{r}_{O} \)

\[ \mathbf{\dot{\mathbf{\omega}}} = \frac{\mathbf{\dot{\omega}}}{\text{sec}^2} = -\left( \frac{4\pi}{\text{sec}^2} \right)^2 \mathbf{k} \]

\[ = \frac{2\pi}{\text{sec}^2} \mathbf{\dot{\omega}} - \frac{8\pi^3}{\text{sec}^2} \mathbf{k} \]

\[ \mathbf{\dot{a}}_{G} = \frac{2\pi}{\text{sec}^2} \mathbf{\dot{\omega}} = \frac{8\pi^3}{\text{sec}^2} \mathbf{k} \]

d) \( \mathbf{\ddot{F}} = \mathbf{M}_{\ddot{a}} \)

\[ \mathbf{F}_0 = \mathbf{M}_{\ddot{a}} = \frac{2\pi ml^2}{\text{sec}^2} \left( 5 - 4\pi^2 \right) \]

e) \( \mathbf{\ddot{N}} = \mathbf{\ddot{T}}_0 \)

\[ \mathbf{T}_0 \mathbf{k} = \frac{\mathbf{\ddot{N}}}{\text{sec}^2} \]

\[ \mathbf{M}_0 \mathbf{k} = \frac{4\pi ml^2}{\text{sec}^2} \frac{4\pi}{\text{sec}^2} \mathbf{k} \]

\[ \Rightarrow \mathbf{M}_0 = \frac{4\pi ml^2}{3} \text{ sec}^2 \]

f) \( \mathbf{\dot{P}}_{\text{in}} = \mathbf{\dot{M}}_0 \mathbf{\dot{w}} = \mathbf{M}_0 \mathbf{k} \cdot \mathbf{\dot{w}} \mathbf{k} = \mathbf{M}_0 \mathbf{\dot{w}} \)

\[ \mathbf{\dot{P}}_{\text{in}} = \frac{4\pi ml^2}{3} \text{ sec}^2 \left( \frac{4\pi}{\text{sec}^2} \right) = \frac{16\pi^3 ml^2}{3} \text{ sec}^3 \]
(b) \[ I_{zz} = \left( I_{zz}^0 \right)_1 + \left( I_{zz}^0 \right)_3 \]

\[ I_{zz}^0 = \frac{2}{3} M R^2 \]

\[ I_{zz} = 2M \left( \frac{2}{3} R^2 + 2^2 \right) \]

(c) For dumbbell, AMBo

For small mass, LMB

\[ \sum M_0 \frac{\hat{a}}{u} = \hat{b}_0 \]

\[ \overrightarrow{F}_{16} \times (-Mg \hat{j}) + \overrightarrow{F}_{16} \times (-Mg \hat{j}) + \overrightarrow{P}_{16} \times -T \hat{i} = \sum M_0 \frac{\hat{a}}{u} \]

\[ \{ Mg \hat{e} - Mg \hat{e} \cos \theta \hat{k} - Ta \hat{k} \} \hat{k} = 2M \left( \frac{2}{3} R^2 + 2^2 \right) a \hat{k} \]

\[ \{ 2 \cdot \hat{k} \} \hat{k} = -Ta = 2M \left( \frac{2}{3} R^2 + 2^2 \right) \alpha \]

LMB

\[ \sum \hat{F} = m \sum \hat{a} = mab \]

\[ \{ \hat{F} + \hat{T} - Mg \hat{j} \} \hat{j} = mab \hat{3} \hat{j} \Rightarrow \hat{T} - Mg = mab \]

Need 1 more equation to solve

2 equations, 3 unknowns T, d, ab
kinematic constraint \[ \alpha_0 = \frac{d\alpha}{dt} \] (iii)

acceleration of mass

\[ T = m(g + \alpha_0) \] (iv) \[ \alpha = \frac{g\alpha_0}{a} \] (v)

Substitute (iv), (v) into (i)

\[-m(g + \alpha_0): \alpha = \frac{z \alpha_0}{\gamma} \left( \frac{3}{5} \frac{R^2 + \ell^2}{a^2} \right) \]

Solve for \( \alpha_0 \):

\[ \alpha_0 = \frac{-g}{\left(1 + \frac{2M}{m} \left( \frac{3}{5} \frac{R^2 + \ell^2}{a^2} \right) \right)} \]
b.) \[ \text{AMB}_q \ (\text{brake}) \ \Sigma \ T = M_v = \frac{\ddot{v}}{\dot{\theta}} \ \vec{Q} \ (M=0) \]
\[ \{N(\ddot{d}+uN) - \dot{P}L\} \ \vec{k} = \vec{0} \]
\[ \{3\} \ \vec{k} \Rightarrow N = \frac{P\dot{L}}{d+uN} \]

\[ \text{AMB}_0 \ (\text{cylinder}) \ \Sigma \ T = M_v = \frac{\ddot{v}}{\dot{\theta}} \]
\[ \{\text{UNR} \vec{k} \ i \times \dot{\vec{k}} \}
\[ \text{mr}\vec{R}\vec{k} \text{ for disk} \]
\[ \{3\} \ \vec{k} \Rightarrow a = \frac{2uN}{MR} \]
\[ \theta = 2 \frac{uP L}{mR(d + uh)} \] 

**Subst. \( \theta \rightarrow \theta' \)**

**Ans. \( \theta = \text{const.} \)**

b) \( \alpha = \frac{d\omega}{dt} = \frac{2uPL}{mR(c + uh)} \)

\[ \Rightarrow \int_{\omega(0)}^{\omega} d\omega = \int_{0}^{t} \frac{2uPL}{mR(c + uh)} dt \]

\[ \Rightarrow \omega - \omega(0) = \frac{2uPL}{mR(c + uh)} t \]

\[ \omega(t) = -\omega(0) + \frac{2uPL}{mR(c + uh)} t \]

When cylinder comes to rest at \( t = t_f \)

\( \omega(t_f) = 0 \Rightarrow \)

\[ 0 = -\omega(0) + \frac{2uPL}{mR(c + uh)} t_f \]

\[ \Rightarrow t_f = \frac{m\omega(0)R(c + uh)}{2uPL} \]

c) **LMB (cylinder)**

\[ \bar{F}_0 - uN\vec{e} - N\vec{S} = m\ddot{\theta}_0 \]

\[ \Rightarrow \bar{F}_0 = \bar{N}(\vec{u}\vec{e} + \vec{S}) \]

\[ \bar{F}_0 = \frac{PL}{c + uh}(\vec{u}\vec{e} + \vec{S}) \]