1. Find the value of $k$ that make the function $f(x) = \frac{k}{x^2}$ on $[2, 6]$ a probability density function.

2. Given the probability density function $f(x) = \frac{1}{30} x$ on $[2, 8]$, determine each of the given probabilities: a. $P(3 < X < 4)$ b. $P(X > 5)$ c. $E(X)$

3. If $X$ has probability density function $f(x) = \frac{1}{8}$ on $[2, 10]$, find each probability:
   a. $P(3 < X < 7)$  
   b. $P(X < 8)$  
   c. Find the mean, variance and standard deviation for the probability density function. (Note: this is a uniform distribution!)

4. The amount of time between looking at a smoke detector and seeing a flash is uniformly distributed on $[0, 20]$.
   a. Find a pdf for the random variable $x$ = amount of time to see a flash.  
   b. What is the probability you will see a flash within 5 seconds of looking at the smoke detector?  
   c. What is the probability you will see a flash between 8 and 16 seconds from looking at the smoke detector?  
   d. Find the expected amount of time you would have to wait to see a flash on the smoke detector.

5. The number of days required for a worker to learn a new skill is a random variable $X$ with probability density function $f(x) = \frac{1}{4} x - \frac{3}{64} x^2$ on $[0, 4]$.
   a. Find the probability it will take 2 or more days to learn the skill.  
   b. Find the expected number of days to learn the skill.  
   c. Find the variance for the number of days to learn the skill.

6. The length of a phone call ($x$ minutes) is given by $f(x) = .25e^{-25x}$, $x \geq 0$.
   a. Find the probability a phone call lasts 3 minutes or less.  
   b. Find the probability that a phone call will take more than 6 minutes.  
   c. Find the expected amount of time for a phone call.

7. Find the expected value of the probability density functions:
   a. $f(x) = 1 - \frac{1}{\sqrt{x}}$ on $[1, 4]$  
   b. $f(x) = 2x^{-3}$ on $[1, \infty)$

8. The annual rainfall in Maine is a random variable with probability density function $f(x) = \frac{1}{15} \left( x + \frac{1}{2} \right)$ on $[0, 5]$. Find the mean rainfall. (Hint: multiply the function out first!)

9. Find the median of each pdf: a. $f(x) = \frac{1}{6\sqrt{x}}$ on $[1, 16]$  
   b. $f(x) = \frac{3}{x^2}$ on $[2, 6]$. 
10. A florist’s daily revenue from the sale of \( b \) bouquets of flowers and \( p \) potted plants can be described by the function \( R(b, p) = 40b + 30p - b^2 - bp \) dollars. What is the revenue for the florist when 20 bouquets of flowers and 5 potted plants are sold?

11. Find the indicated partial derivatives.
   
   a. \( f(x, y) = 5x^3y^2 + 3xy + 5y^2 \), Find \( f_x \)  
   
   b. \( f(x, y) = 3y^4e^{2x} \), Find \( f_y \)  
   
   c. \( f(x, y) = \ln(2x + 3y) \), Find \( f_x \)  
   
   d. \( f(x, y) = 4(2x^2 - 5y^2)^3 \), Find \( f_y \)  
   
   e. \( f(x, y) = 4e^{2xy} \), Find \( f_x \)  
   
   f. \( f(x, y) = \frac{5}{(3x^2 - 7xy)^6} \), Find \( f_x \)  
   
   g. \( f(x, y) = 4x^4y^3 - 2x^3y^2 - 6x^4y \), Find \( f_{xx} \)  
   
   h. \( f(x, y) = \frac{3x^4}{y^5} \), Find \( f_{yy} \)  

12. Determine the critical point(s) for the function \( f(x, y) = 5y^3 + x^2 - 4x - 240y - 14 \). Classify each point as a relative maximum, relative minimum or saddle point.

13. A bookstore’s daily revenue from the sale of \( s \) sweatshirts and \( t \) T-shirts can be modeled by \( R(s, t) = 40s - s^2 + 50t - t^2 - st \) dollars. Find the values of \( s \) and \( t \) that maximize the daily revenue.

14. A botanist finds that the growth rate of a particular plant is a function of the temperature and the humidity of the greenhouse. After a great deal of experimentation the botanist concludes that the growth rate \( g(x, y) \) in inches per week when the greenhouse has a temperature of \( x \) degrees F and humidity of \( y \) percent is given by \( g(x, y) = -2x^2 + 196x - 3y^2 + 242y + 2xy - 16360 \). At what temperature and humidity should the greenhouse be kept to maximize the growth rate of the plant? How fast will the plant grow under these optimal conditions?

15. Evaluate each double integral.
   
   a. \( \int_{0}^{2} \int_{1}^{6} 6x^2y \, dx \, dy \)  
   
   b. \( \int_{3}^{6} \int_{0}^{2} (3x + 2y) \, dx \, dy \)  
   
   c. \( \int_{0}^{2} \int_{1}^{4} 4xy \, dy \, dx \)  

16. a. \( \int_{0}^{3} \int_{1}^{2} (x + y) \, dy \, dx \)  
   
   b. \( \int_{-3}^{3} \int_{0}^{2} y^2e^{-x} \, dy \, dx \)  
   
   c. \( \int_{1}^{2} \int_{0}^{x+1} (x - 2y) \, dy \, dx \)
Key
1. k = 3
2. a. 7/60  b. 13/20 = .65  c. 5.6
3. a. 1/2  b. 3/4  c. mean is 6, variance is 16/3 = 5.33, st. deviation = 2.31
4. a. f(x) = 1/20, [0, 20]  b. 5/20 = .25  c. 8/20 = .4  d. 10 seconds
5. a. .625 = 5/8  b. 2 1/3 days  c. 6.4 – (2.3)^2 = 1.11
6. a. 0.5276  b. 0.2231  c. 4 minutes
7. a. 2.83  b. 2
8. 3.194 inches
10. When 20 bouquets of flowers and 5 potted plants are sold the daily revenue will be $450.
11. a. \( f_x = 15x^3y^2 + 3y \)  b. \( f_y = 12y^3e^{2x} \)
    c. \( f_x = \frac{2}{2x+3y} \)  d. \( f_y = -120y(2x^2 - 5y^2)^2 \)
    e. \( f_x = 8ye^{2xy} \)  f. \( f_x = -30(6x - 7y)(3x^2 - 7xy)^{-7} \)
    g. \( f_{xx} = 48x^2y^5 - 12xy^2 - 120x^3y \)  h. \( f_{yy} = 90x^4y^{-7} \)
12. Relative Minimim at (2, 4) and saddle point at (2, -4)
13. When 10 sweatshirts and 20 t-shirts are sold the daily revenue is maximized.
14. Keeping the greenhouse at a temperature of 83 degrees F and 68% humidity will give a growth rate of 2 inches per week.
15. a. 112  b. 96  c. 147
16. a. 12  b. \(-9e^{-3} + 9e^3\)  c. -2.5