1. Find the value of k that makes the function \( f(x) = \frac{x^3}{k} \) on \([0, 2]\) a probability density function.

2. Given the probability density function \( f(x) = \frac{2}{15} x \) on \([1, 4]\), determine each of the given probabilities: a. \( P(2 < X < 3) \) b. \( P(X > 1.5) \)

3. If \( X \) has probability density function \( f(x) = \frac{1}{10} \) on \([0, 10]\), find each probability:
   a. \( P(3 < X < 7) \) b. \( P(X < 8) \) c. Find the mean, variance and standard deviation for the probability density function. (Note: this is a uniform probability distribution!)

4. Let \( W \) be a random variable with Cumulative Distribution Function \( F(x) = \frac{1}{27} x^3 \), \([0, 3]\).
   a. Find \( P(W < 1.5) \) b. Find \( P(1 < W < 2) \) c. Find \( P(W > .75) \)

5. The number of days required for a worker to learn a new skill is a random variable \( X \) with probability density function \( f(x) = \frac{4}{9} x^2 - \frac{4}{27} x^3 \) on \([0, 3]\).
   a. Find the probability it will take 2 or more days to learn the skill.
   b. Find the expected number of days to learn the skill.
   c. Find the variance for the number of days to learn the skill.

6. The waiting time in hours at an auto muffler shop is a random variable \( X \) with a probability density function given by \( f(x) = \frac{3}{2} x - \frac{3}{4} x^2 \) on \([0, 2]\) hours. Find the probability that you will have to wait
   a. at most 1 hour b. between \( \frac{1}{2} \) and \( 1 \frac{1}{2} \) hours c. Find the expected waiting time.

7. Psychologists have determined that the time it takes a rat to find its way through a maze is exponentially distributed. Let \( x \) be the time in seconds, and for a particular maze the probability density function is \( f(x) = 0.025e^{-0.025x} \) seconds, \( x \geq 0 \). (Note: an exponential pdf)
   a. Find the probability that a rat can find its way through the maze in 20 seconds or less.
   b. Find the probability that a rat will take between 30 seconds and 1 minute (60 seconds) to get through the maze.
   c. Find the probability that a rat will take at least 80 seconds to find its way through the maze.
   d. Find the expected amount of time it will take the rat to get through the maze.
8. Find the expected value of the probability density functions:
   a. \( f(x) = 1 - \frac{1}{\sqrt{x}} \) on \([1, 4]\)  
   b. \( f(x) = 4x^{-5} \) on \([1, \infty)\)

9. The annual rainfall in Maine is a random variable with probability density function
   \( f(x) = \frac{1}{15} \left( x + \frac{1}{2} \right) \) on \([0, 5]\). Find the mean rainfall. (Hint: Multiply the expression out before you integrate.)

10. The life span of a Karstens Beetle is uniformly distributed on \([2, 5]\) years.
   a. Find the probability a Karstens Beetle live longer than 3 years.
   b. Find the mean lifespan for the Karstens Beetle.

11. Find the median for the probability density function \( f(x) = \frac{x}{8} - \frac{1}{2} \) on \([4, 8]\).

Key
1. \( k = 4 \)
2. a. 1/3  
   b. 55/60
3. a. 4/10  
   b. 8/10  
   c. mean is 5, variance is 100/12 = 8.33, st. deviation = 2.89
4. a. 0.125  
   b. 7/27  
   c. 0.984375
5. a. 11/27  
   b. 1.8 days  
   c. \( 3.6 - (1.8)^2 = 0.36 \)
6. a. 0.50  
   b. 0.6875 = 11/16  
   c. Expected wait time of 1 hour
7. a. 0.3935  
   b. 0.2493  
   c. 0.1353  
   d. 40 seconds
8. a. 2.83  
   b. 1.33
9. 3.194 inches
10. a. 2/3  
    b. 3.5 years
11. 6.83