1) Integrate.
   a) \( \int 4x^6 \, dx = 4x^7 + C \)
   b) \( \int \frac{6}{x} \, dx = 6 \ln |x| + C \)
   c) \( \int \sin(3x) \, dx = -\frac{1}{3} \cos(3x) + C \) *Remember the integration factor*
   d) \( \int \frac{8}{(4x+1)^3} \, dx = \frac{1}{8} \left( \frac{1}{4x+1} \right)^{-2} = \left( \frac{1}{4x+1} \right)^{-2} + C \)
   e) \( \int \sqrt{1-x} \, dx = \frac{2}{3} (1-x)^{3/2} + C \)
   f) \( \int x^5 (x^9 + 1)^5 \, dx = \frac{1}{5} \int (x^9 + 1)^6 \, du = \frac{1}{5} (x^9 + 1)^6 + C \)
   g) \( \int \cos x e^{\sin x} \, dx = \int e^u \, du = e^u + C = e^{\sin x} + C \)

average value
- probability
- DEq \( \rightarrow \) separable variables
- Double Integrals
- Improper Integrals
2) Find the value of \( k \), \[ \frac{d}{dx} \left. \frac{x^2 + 4x}{1} \right|_{1}^{x} \]
a) \[ \int 2x + 4 \, dx = 2.7 \] 
\[ k^2 + 4k - (1^2 + 4(1)) = k^2 + 4k - 5 = 2.7 \]
\[ -27 - 2.7 \]
\[ k^2 + 4k - 32 = 0 \]
\[ (k + 8)(k - 4) = 0 \]
\[ k = -8, 4 \]

3) \( \int \frac{2x}{e^{4x}} \, dx \)
\[ u = D, \quad dv = \frac{1}{e^{-4x}} \]
\[ 2x + \frac{2x}{4} = -4x \]
\[ -\frac{2x}{4} e^{-4x} \]
\[ \frac{2}{16} e^{-4x} + C \]
\[ = \frac{1}{2} x e^{-4x} - \frac{1}{8} e^{-4x} + C \]

b) \[ \int 4x^3 \ln x \, dx \]
\[ \ln x + \frac{dx}{4x^3} \]
\[ \frac{1}{x} \ln x - \int \frac{1}{x} \, dx \]
\[ \frac{1}{x} (x^2) = x^3 \]

4) Find the area between the curves, \( y = 14 - 3x \) and \( y = x + 6 \) from \( x = 0 \) to \( x = 4 \)
\[ 14 - 3x = x + 6 \]
\[ 14 = 4x + 6 \]
\[ 4x = 8 \]
\[ x = 2 \]
\[ \int_{0}^{4} (14 - 3x - (x + 6)) \, dx + \int_{2}^{4} (x + 6 - (14 - 3x)) \, dx \]
\[ = -2x^2 + 8x \bigg|_{0}^{2} + 2x^2 - 8x \bigg|_{2}^{4} \]
\[ = -2(2)^2 + 8(2) - 0 \]
\[ = 2(4)^2 - 8(4) - (2(2)^2 - 8(2)) \]
\[ = 32 - 32 - (8 - 16) \]
\[ = 8 \]
5. Average Value

a) \[ f(x) = \frac{4}{(2x+1)^2} \text{ on } [0, 2] \]

\[
\frac{1}{2} \int_0^2 4(2x+1)^{-2} \, dx = \frac{1}{2} \left[ \frac{4}{2} (2x+1)^{-1} \right]_0^2 = \frac{1}{2} \left( \frac{-2}{2} \right)_{-1}^{\frac{2}{3}}
\]

\[
= \frac{1}{2} \left( \frac{-2}{2} + \frac{2}{3} \right) = \frac{1}{2} \left( \frac{-2}{6} + \frac{4}{6} \right) = \frac{1}{2} \left( \frac{2}{6} \right) = \frac{1}{2} \left( \frac{2}{3} \right) = \frac{1}{3}
\]

\[
= \left[ 4e^{2x} \right]_0^2 = 4e^{(2)(2)} - 4e^{(2)(0)} = 4e^4 - 4e^0 = 4e^4 - 4
\]

b) \[ f(x) = 4e^{2x} \text{ on } [0, 5] \]

\[
\frac{1}{5} \int_0^5 4e^{2x} \, dx = \frac{1}{5} \left[ \frac{4}{2} e^{2x} \right]_0^5 = \frac{1}{5} \left[ 2e^{10} - 2 \right]
\]

\[
= \frac{2}{5} e^{10} - \frac{2}{5} = \frac{2}{5} \left( e^{10} - 1 \right) = \frac{2}{5} \left( e^{10} - 1 \right)
\]

6. \[ \int_4^\infty \frac{2}{x^4} \, dx \]

\[
= \left. \frac{-2}{3} x^{-3} \right|_4^\infty = \frac{-2}{3} \left( \frac{1}{\infty} - \frac{1}{4} \right) = \frac{-2}{3} \left( 0 - \frac{1}{4} \right) = \frac{1}{6}
\]

\[
= \frac{2}{192} = \frac{1}{96}
\]

b) \[ \int_0^\infty 2e^{-1x} \, dx \]

\[
= \left. \frac{-2}{e^{1x}} \right|_0^\infty = \frac{-2}{e^\infty} + \frac{20}{e^0} = \frac{-2}{\infty} + \frac{20}{1} = 20
\]
7) Separable Variables

a) \( \frac{1}{y} \frac{dy}{dx} = \frac{6 \sqrt{x}}{y} \) \( y(6) = 8 \rightarrow C \)

\[
S \frac{1}{y} \frac{dy}{dx} = S 6 \sqrt{x} \ dx \\
S y \frac{dy}{dx} = S 6 \sqrt{x} \ dx \\
\ln y = \frac{2}{3} x^{3/2} + C \\
\ln y_6 = \frac{2}{3} \cdot 6^{3/2} + C \]

\[
y = Ce^{\frac{2}{3} x^{3/2}} \\
y_6 = Ce^{\frac{2}{3} \cdot 6^{3/2}} \\
y_6 = 8 \\
8 = Ce^{4(\frac{3}{2})} \\
C = e^{-4} \]

b) \( e^y \frac{dy}{dx} = \cos x \frac{e^y}{e^y} \)

\[
S e^y \frac{dy}{dx} = S \cos x \ dx \\
S e^y \ dx = S \cos x \ dx \\
\ln(e^y) = \ln(\sin x + C) \]

\[
y = \ln(\sin x + C) \quad \text{yes!} \]

\[
\text{not } y = \ln(\sin x) + C \]

8) Time to complete a task is a pdf w/ pdf not a pdf.

f(x) = \frac{1}{2} x - \frac{1}{8} \text{ on } [0, 10].

a) Find prob. of completing the task \( x \leq 2 \) min.

\[
S \left( \frac{1}{2} x - \frac{1}{8} \right) \ dx = \left[ \frac{1}{4} x^2 - \frac{1}{8} x \right]_0^2 = \frac{1}{4} (2)^2 - \frac{1}{8} (2) \]

\[
= \frac{1}{4} - \frac{1}{8} = \frac{2}{8} \]

b) Find Expected time to complete the task, \( x \leq 4 \)

\[
S x \left( \frac{1}{2} x - \frac{1}{8} \right) \ dx = \left[ \frac{1}{4} x^2 - \frac{1}{8} x \right]_0^4 = \frac{1}{4} \left( \frac{1}{8} \right)^2 - \frac{1}{8} \cdot 4 \\
= \frac{1}{6} - \frac{16}{16} = 10 \cdot 2 - 1 = 9 \]
9) Double Integrals
\[
\int_0^2 \left( \int_0^1 2xy + 4 \, dx \right) \, dy = \int_0^2 3y + 4 \, dy
\]
\[
= \left. \frac{3}{2} y^2 + 4y \right|_0^2 = \frac{3}{2} (2)^2 + 4(2) - 0
\]
\[
= 2^2 y + 4(2) - (1^2 y + 4(1)) = \frac{12}{2} + 8
\]
\[
= 14y + 8 - y - 4 = 3y + 4 = 14
\]

10) Partials
\[f(x, y) = H e^{2x^2y^3}\]
\[f_x = 4e^{2x^2y^3} 4xy^3 \quad f_y = 4e^{2x^2y^3} 2x^2(3y^2)\]
\[b) f(x, y) = \ln(6x - x^2y)\]
\[f_x = \frac{6 - 2xy}{6x - x^2y} \quad f_y = \frac{-x^2}{6x - x^2y}\]

\[f(x, y) = \frac{2}{3} x^3 + \frac{1}{2} x^2 - 10x + \frac{4}{2} y^2 - 18y\]
\[f_x = 2x^2 + x - 10 \quad \quad (2x + 5)(x - 2) = 0 \quad x = \frac{-5}{2}, 2\]
\[f_y = 9y - 18 \quad \quad 9y - 18 = 0 \quad 9y = 18 \quad y = 2\]
\[9y = 18 \quad y = 2\]

12) Annuities
\[S_n = R \left[ \frac{(1 + \frac{R}{m})^{nt} - 1}{\frac{R}{m}} \right] \approx 10,000 \left[ \frac{(1 + \frac{0.06}{12})^{48} - 1}{\frac{0.06}{12}} \right]\]
\[10,000 \text{ for 4 yrs, } \text{690 monthly} \]