The rate of change of a quantity is proportional to the amount present at time \( t \) is represented by the differential equation \( \frac{dA}{dt} = kA \). The solution of this differential equation is \( A = Me^{kt} \). The constant \( k \) is called the growth constant, while \( M \) is the amount present at \( t = 0 \), often called the initial condition. A positive value of \( k \) indicates growth and a negative \( k \) indicates decay.

1. Sean deposits $3000 in an account that increases at a rate of 4% (compound continuously). Find the value of the deposit after 8 years.

   \[
   \frac{dA}{dt} = 0.04A \\
   A = Me^{0.04t} \\
   A(0) = 3000 \\
   A(8) = 3000e^{0.04(8)} = 4131.38
   \]

   Find the value of the deposit after 8 years.

2. Sales at Augieburger's Grill are increasing at a rate proportional to the amount of sales, with a growth rate of 8%. How many years will it take for sales to triple at Augieburger's?

   \[
   \frac{dS}{dt} = 0.08S \\
   S(t) = Me^{0.08t} \\
   M = \text{initial sales} \to 100 \\
   \text{triple sales} \to 3M \to 300
   \]

   Find \( t \) when \( S(t) = 3M \)

   \[
   \ln(3) = 0.08t \\
   t = \frac{\ln(3)}{0.08} = 13.73 \text{ yrs.}
   \]
3. Sales (in thousands) of a certain product are declining at a rate proportional to the amount of sales, with a decay rate of 10% per year. How much time will pass before sales are half of their original value?

\[ K = -0.10 \]

\[ \frac{dS}{dt} = -0.10S \]

\[ S = Me^{-0.10t} \]

\[ S(t) = Me^{-0.10t} \]

\[ \frac{1}{2}M = \frac{Me^{-0.10t}}{M} \]

\[ ln(0.5) = -0.10t \]

\[ \frac{ln(0.5)}{-0.10} = t \]

\[ t = 6.93 \text{ yrs} \]

4. Profits at Garrett’s Galleria are predicted to increase from 2 (million) in 2010 to 6 (million) in 2012.

a. Assuming unlimited growth and the growth model \( \frac{dy}{dt} = ky \) fits this situation, find the value of the growth constant \( k \).

b. What is the profit after 4 years?

c. When will Garrett have a profit of 20 (million)?

\[ 2 \text{ (million) in 2010} \quad (t=0) \]

\[ 6 \text{ (million) in 2012} \quad (t=2) \quad \Rightarrow \quad \text{to find } k \]

\[ y(t) = Me^{kt} \]

\[ y(t) = 2e^{kt} \]

\[ 6 = 2e^{k(2)} \]

\[ \ln(3) = e^{k(2)} \]

\[ \ln(3) = \frac{2k}{2} \]

\[ k = \frac{\ln(3)}{2} \approx 0.5493 \]

\[ 0.5493t \]

\[ y(t) = 2e^{0.5493t} \]

\[ y(4) = 2e^{0.5493 \times 4} \]

\[ y(4) = 17,999.5 \text{ million} \]

\[ (\text{not 17,999,500}) \]

\[ \text{(or 18,000,000)} \]

(\[ \text{Find } y \text{ when } t=4 \]

\[ y(t) = 2e^{0.5493t} \]

\[ y(4) = 17,999.5 \text{ million} \]

\[ \text{18 million} \]

\[ \text{(or 18,000,000)} \]

\[ \text{Find } t \text{ when } y(t) = 20 \]

\[ ln(10) = ln(e^{0.5493t}) \]

\[ 20 = \frac{2e^{0.5493t}}{2} \]

\[ t = \frac{ln(10)}{0.5493} \approx 4.19 \text{ yrs} \]