Math 19 Test 2 Review

1. Each day, the total output of a coal mine after $t$ hours of operation is approximately

$$o(t) = 40t + t^2 - \frac{1}{15}t^3$$ tons, for up to 12 hours of operation.

a. What is the average rate of change in the output of coal when the mine operates between 5 and 8 hours?

b. What is the rate of output of coal at $t = 5$ hours?

2. Liquid is pouring into a large vat. After $t$ hours, there are

$$g(t) = 5t - \sqrt{t}$$ gallons in the vat. At what rate is the liquid pouring into the vat at 4 hours?

3. After an advertising campaign, the sales of a product often increase and then decrease. Suppose that $t$ days after the end of the advertising, the daily sales are

$$f(t) = -3t^2 + 32t + 100$$ units. At what rate are sales changing 2 days after the end of advertising?

4. Solve each problem about the tangent line of a function.

a. Find the slope of the tangent line to the graph of $y = 4x^{3/2} - 5x^{1/2}$ when $x = 16$.

b. Find an equation for the tangent line to the graph of $y = x - x^2$ when $x = -3$.

c. Find all values of $x$ (if any) where the tangent line to the graph of the

$$y = x^3 + 2x^2 - 20x + 10$$ is horizontal.

d. Find all values of $x$ (if any) where the tangent line to the graph of the

$$y = \frac{x^2 + 5}{2x^2 - 1}$$ is horizontal.

5. A manufacturer estimates that the hourly cost of producing $x$ units of a product on an assembly line is

$$C(x) = 0.1x^3 - 6x^2 + 136x + 200$$ dollars. Find the marginal cost when 20 units are produced.

6. A helicopter is rising straight up. While in the air, the pilot drops a pair of binoculars. The position (height) of the binoculars is given by

$$s(t) = -16t^2 + 32t + 128$$ feet at $t$ seconds. How long (how many seconds) will it take for the binoculars to hit the ground (when is $s(t) = 0$)? What is the velocity of the binoculars when they smash into the ground?

7. Differentiate.

a. $f(x) = 3\sqrt{x} - \frac{3}{\sqrt{x}}$  
   b. $f(x) = 4\sqrt{x^2 + 8}$  
   c. $f(x) = \frac{5x}{2} - \frac{2}{5x}$

d. $f(x) = \frac{4}{5x^2}$  
   e. $f(x) = \frac{x-6}{2x+7}$  
   f. $f(x) = 3x^5 - 6x^3 - 8$
g. \( f(x) = \frac{10}{(3x+8)^4} \)  

h. \( f(x) = 6x^{2/3} \)  

i. \( f(x) = (4x+3)^3(3x+1)^5 \)  

8. Let \( f(x) = \begin{cases} x+6, & x > 3 \\ x^2, & x \leq 3 \end{cases} \) and evaluate each of the following:

a. \( f(3) \)  

b. \( \lim_{x \to 3^-} f(x) \)  

c. \( \lim_{x \to 3^+} f(x) \)  

d. \( \lim_{x \to 3^-} f(x) \)  

9. Let \( f(x) = \begin{cases} x, & x > 3 \\ \frac{x}{6}, & x \leq 3 \end{cases} \) and evaluate each of the following:

a. \( f(3) \)  

b. \( \lim_{x \to 3^-} f(x) \)  

c. \( \lim_{x \to 3^+} f(x) \)  

d. \( \lim_{x \to 3^-} f(x) \)  

10. Evaluate \( \lim_{x \to 4} \frac{x^2 - 3x - 4}{x - 4} \)  

11. Evaluate \( \lim_{x \to \infty} \frac{11x}{5x - 2} \)  

KEY  

1. a. 44.4 tons/hour  
   b. 45 tons/hour  
2. 4.75 gallons/hour  
3. 20 units/day  
4. a. 187/8  
   b. \( y = 7x + 9 \)  
   c. \( x = -10/3, 2 \)  
   d. \( x = 0 \)  
5. $16/unit  

6. The binoculars will hit the ground at 4 seconds. The velocity at that time will be -96 feet/second.  

7. a. \( f'(x) = \frac{3}{2}x^{-1/2} + \frac{3}{2}x^{-3/2} \)  
   b. \( f'(x) = 4x(x^2 + 8)^{-1/2} \)  
   c. \( f'(x) = \frac{5}{2} + \frac{2}{5}x^{-2} \)  
   d. \( f'(x) = -\frac{8}{5}x^{-3} \)  
   e. \( f'(x) = \frac{19}{(2x+7)^2} \)  
   f. \( f'(x) = 15x^4 - 18x^2 \)  
   g. \( f'(x) = \frac{-120}{(3x-8)^5} \)  
   h. \( f'(x) = 4x^{-1/3} \)  
   i. \( f'(x) = 12(4x+3)^2(3x+1)^5 + 15(4x+3)^3(3x+1)^4 \)  

8. a. 9  
   b. 9  
   c. 9  
   d. 9  

9. a. 14  
   b. 14  
   c. \( \frac{1}{2} \)  
   d. DNE  

10. 5  
11. 11/5