1. Michael and Jonathan are studying a group of iguanas that reside in the Painted Desert. The population can be described by $P(t) = 0.5x^2 - 0.25x + 30$ iguanas, $t$ years after 2005.
   a. Find the average rate of change in the number of iguanas between 2005 and 2010.
   b. Find the rate of change in the number of iguanas in the year 2006.

2. Find the derivative for each of the following:
   a. $f(x) = 4x^3 - 5x$
   b. $f(x) = 2\sqrt{4x+1}$
   c. $f(x) = 3e^{4x}$
   d. $f(x) = \frac{3}{x^2}$
   e. $f(x) = 5\ln(1 + x)$
   f. $f(x) = 2\sin(3x + 4)$
   g. $f(x) = 3xe^x$
   h. $f(x) = \frac{4}{(3x + 2)^3}$
   i. $f(x) = 3(\sin 2x)^5$
   j. $f(x) = \frac{e^{4x}}{x^3}$
   k. $f(x) = \cos(3x - 7)$
   l. $f(x) = \ln(\sin x)$

3. Evaluate each limit.
   a. $\lim_{x \to 1} x + 1$
   b. $\lim_{x \to -2} \frac{x^2 + x - 2}{x + 2}$
   c. $\lim_{x \to \infty} \frac{x^2 + 6}{3x^2}$

4. Use the graph below to decide whether the following limits exist. If a limit exists, give its value.

5. Andy and Art own Sports Photography Plus, which specializes in 5 x 7 inch action shots of local athletes. Costs for each photo have been calculated to be $C(x) = 0.02x^2 + 0.5x + 10$ dollars for each photo. The photos sell for $20 each.
   a. What is the marginal cost when 10 photos are made?
   b. What is the profit when 10 photos are bought and sold?
   c. What is the marginal profit for 10 photos?
6. Use the graph of the function $f(x)$ given below to answer the following questions about the derivative $f'(x)$.

7. Find all $x$ values for which the slope of the tangent line to the curve $y = x^3 + 9x^2 + 6x$ at the given $x$ value is 6.

8. Find the open intervals where the function $f(x) = xe^{-5x}$ is increasing. Give your answer(s) in interval notation. If there are no intervals, then write NONE.

9. Consider the function $f(x) = \ln(2x^2 + 1)$.
   a. Find the second derivative for the function.
   b. Find the $x$ values of the two inflection points.

10. Consider the function $f(x) = x^4 - 8x^3 + 17$
    a. Find the relative minimum for the function.
    b. The function has 2 inflection points, find each of them.

11. Find the absolute maximum and minimum for the function $f(x) = x^3 + 3x^2 - 9x + 2$ on $[0, 2]$.

12. A farmer has 100 feet of fencing and wants to build a rectangular enclosure along a straight wall. If the side along the wall needs no fence, find the dimensions that make the enclosure as large as possible. Also find the maximum area.

13. A homeowner wants to build a garden surrounded by a fence along his driveway. If the garden is to be 800 square feet and the fence along the driveway costs $6 per foot whereas on the other three sides it costs on $2 per foot, find the dimensions of the garden that will minimize the cost. What is the minimum cost?

14. Find $\frac{dy}{dx}$ for $y^3 + 2y - x^4 = 10$ using implicit differentiation.

15. Find $\frac{dy}{dx}$ for $x^2 y^2 - 3x = 8$
16. Evaluate \( \frac{dy}{dt} \) for \( x^3 + 4x = 3y^4 - 2y \) if \( \frac{dx}{dt} = 3, \ x = -1, \ y = \frac{1}{2} \).

17. Evaluate \( \frac{dy}{dt} \) for \( 5x^2 + 2x^3 y - 6y^4 = 7 \) if \( \frac{dx}{dt} = 3, \ x = -1, \ y = \frac{1}{2} \).

18. The area of a circular sunspot is growing at a rate of 1200 sq. km/sec. How fast is the radius growing when it equals 10,000 km?

19. Find the equation for the tangent line to the curve \( f(x) = x \sin(2x) \) when \( x = \frac{\pi}{4} \).

Key

1. a. The average rate of change in the iguana population between 2005 and 2010 is 2.25 iguanas per year.
   b. The rate of change in the iguana population in 2006 is approximately .75 iguanas/year.

2. a. \( f'(x) = 12x^2 - 5 \) b. \( f'(x) = (4x + 1)^{1/2} \) c. \( f'(x) = 3e^{4x} \) d. \( f'(x) = -6x^3 \) e. \( f'(x) = 5 \left( \frac{1}{1 + x} \right) \) f. \( f'(x) = 6 \cos(3x + 4) \)
   g. Product Rule \( f'(x) = 3xe^x + 3e^x \) h. \( f'(x) = -12(3x + 2)^{-4} \) i. \( f'(x) = 30 \cos(2x)(\sin 2x)^4 \) j. \( f'(x) = \frac{4e^{4x}x^3 - e^{4x}3x^2}{(x^3)^2} \)
   k. \( f'(x) = -3 \sin(3x - 7) \) l. \( f'(x) = \frac{\cos x}{\sin x} \)

3. a. 8 b. -3 c. 1/3

4. a. -1 b. -1 c. 1 d. Does Not Exist (RHL not equal to LHL)

5. a. $0.90/photo b. $183 c. $19.10/photo

6. negative, zero, zero, positive, undefined

7. \( x = 0 \) and \( -6 \)

8. \( (-\infty, 1/5) \)

9. a. \( f''(x) = \frac{-8x^2 + 4}{(2x^2 + 1)^2} \) b. \( x = \pm \sqrt{1/2} \)

10. a. (6, -415) b. (0, 17) and (4, -239)
11. Absolute maximum at (2, 4) and Absolute minimum at (1, -3)

12. Side perpendicular to the wall 25 feet, side parallel to the wall 50 feet. Area 1250 square feet.

13. You will minimize $C = 8x + 4y$ subject to $xy = 800$. Solving you get $x = 20$, $y = 40$ and the cost is $C = 8(20) + 4(40) = $3200

14. $\frac{dy}{dx} = \frac{4x^3}{3y^2 + 2}$

15. $\frac{dy}{dx} = \frac{3 - 2xy^2}{2x^2 y}$

16. $\frac{dy}{dt} = -42$

17. $\frac{dy}{dt} = -\frac{21}{5}$

18. 0.019 km/sec

19. $f'(x) = \sin(2x) + 2x \cos(2x)$ and evaluating at $x = \frac{\pi}{4}$ gives $f'(\pi / 4) = 1$. At $x = \frac{\pi}{4}$, $f(\pi / 4) = \pi / 4$ so use $m = 1$ and the point $(\pi / 4, \pi / 4)$ gives the equation of the tangent line $y = x$. 