2. Mark each statement True or False. Justify each answer.
   (a) Some unbounded sets are compact.
   (b) If \( S \) is a compact subset of \( \mathbb{R} \), then there is at least one point in \( \mathbb{R} \) that is an accumulation point of \( S \).
   (c) If \( S \) is compact and \( x \) is an accumulation point of \( S \), then \( x \in S \).
   (d) If \( S \) is unbounded, then \( S \) has at least one accumulation point.
   (e) Let \( \mathcal{F} = \{ A_i : i \in \mathbb{N} \} \) and suppose that the intersection of any finite subfamily of \( \mathcal{F} \) is nonempty. If \( \bigcap \mathcal{F} = \emptyset \), then for some \( k \in \mathbb{N} \), \( A_k \) is not compact.

3. Show that each subset of \( \mathbb{R} \) is not compact by describing an open cover for it that has no finite subcover.
   (a) \([1, 3)\]
   (b) \([1, 2) \cup (3, 4]\)
   (c) \(\mathbb{N}\)
   (d) \(\{1/n : n \in \mathbb{N}\}\)
   (e) \(\{x \in \mathbb{Q} : 0 \leq x \leq 2\}\)

4. Prove that the intersection of any collection of compact sets is compact.

5. (a) If \( S_1 \) and \( S_2 \) are compact subsets of \( \mathbb{R} \), prove that \( S_1 \cup S_2 \) is compact. \(\checkmark\)
   (b) Find an infinite collection \( \{S_n : n \in \mathbb{N}\} \) of compact sets in \( \mathbb{R} \) such that \( \bigcup_{n=1}^{\infty} S_n \) is not compact.

6. Show that compactness is necessary in Corollary 3.5.8. That is, find a family of intervals \( \{A_n : n \in \mathbb{N}\} \) with \( A_{n+1} \subseteq A_n \) for all \( n \), \( \bigcap_{n=1}^{\infty} A_n = \emptyset \), and such that
   (a) The sets \( A_n \) are all closed.
   (b) The sets \( A_n \) are all bounded.

7. (a) Let \( \mathcal{F} \) be a collection of disjoint open subsets of \( \mathbb{R} \). Prove that \( \mathcal{F} \) is countable. \(\star\)
   (b) Find an example of a collection of disjoint closed subsets of \( \mathbb{R} \) that is not countable.

*8. If \( S \) is a compact subset of \( \mathbb{R} \) and \( T \) is a closed subset of \( S \), then \( T \) is compact.
   (a) Prove this using the definition of compactness.
   (b) Prove this using the Heine–Borel theorem.

9. Find an uncountable open cover \( \mathcal{F} \) of \( \mathbb{R} \) such that \( \mathcal{F} \) has no finite subcover. Does \( \mathcal{F} \) contain a countable subcover?

10. Let \( \mathcal{G} = \{N(p;r) : p \in \mathbb{Q} \text{ and } r > 0\} \).
    (a) Prove that \( \mathcal{G} \) is countable.