Math 124 Quiz #1

The problems have equal credit. Show your work!

1. Below is an augmented matrix for a system of linear equations. Find the system’s general solution (write “inconsistent” if it has no solution).

\[
\begin{bmatrix}
1 & -3 & 1 & 1 & 0 \\
2 & -5 & 2 & 0 & -2 \\
-1 & 4 & -1 & -2 & 3 \\
\end{bmatrix}
\]

2. Like #1, but applied to the matrix

\[
\begin{bmatrix}
1 & -1 & 2 & 3 \\
2 & -3 & 4 & 5 \\
2 & 2 & 4 & 10 \\
\end{bmatrix}
\]

3. Consider the following augmented matrix for a system of linear equations. For what value(s) of \( h \) does the system have: a) a unique solution? b) infinitely many solutions? c) no solution?

\[
\begin{bmatrix}
1 & 2 & h \\
3 & 6 & 9 \\
\end{bmatrix}
\]

4. Like #3, but applied to the matrix

\[
\begin{bmatrix}
2 & 3 & 0 \\
h & 6 & 5 \\
\end{bmatrix}
\]

5. Find a relation (of the form \( \alpha_1 b_1 + \alpha_2 b_2 + \alpha_3 b_3 = 0 \)) that guarantees that the following augmented matrix represents a consistent system.

\[
\begin{bmatrix}
1 & -1 & 2 & b_1 \\
3 & -1 & 4 & b_2 \\
1 & 0 & 1 & b_3 \\
\end{bmatrix}
\]

Math 124 Quiz #2

The problems have equal credit. Show your work!

1. Use linear algebra to balance the following chemical equation (zinc sulfide and oxygen go to form zinc oxide plus sulfur dioxide):

\[
ZnS + O_2 \rightarrow ZnO + SO_2.
\]
2. An economy has two sectors, Rockers and Stoners. Its unit of currency is the rubber ducky (RD). Rockers sell 2/3 of their output to the Stoners and sell the rest to themselves. Stoners sell 3/4 of their output to the Rockers the rest to themselves. The total price of the Stoner output is 240 RD. What is the equilibrium price of the Rocker output?

3. Are the columns of the following matrix linearly independent or linearly dependent? Do they span all of $\mathbb{R}^4$? Justify your answers!

$$\begin{bmatrix}
-1 & 1 & 0 & -1 \\
-2 & 3 & 1 & -1 \\
-3 & 4 & 1 & 1 \\
0 & 1 & 1 & -1
\end{bmatrix}.$$  

4. A linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ satisfies:

$$T\left(\begin{bmatrix} 3 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 3 \end{bmatrix};$$

$$T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}.$$  

Find $T$’s standard matrix.

Math 124 Midterm Exam  
Summer, 2014

The problems have equal credit. Show your work!

1. Determine whether the following matrix is invertible and find the inverse if it exists. You will NOT need fractions to do this problem.

$$\begin{bmatrix}
-1 & 1 & 0 \\
2 & -1 & -1 \\
-1 & 2 & 0
\end{bmatrix}.$$  

2. Like #1, but applied to the matrix

$$\begin{bmatrix}
2 & 3 \\
2 & 7
\end{bmatrix}.$$  

This time you’ll need fractions.

3. Suppose $B$ is an $8 \times 8$ matrix and $Bx = e_8$ is unsolvable. How many solutions does $Bx = 0$ have—or do we need more information to tell? Explain your answer.

4. The same as #3, with one change: now $B$ is an $8 \times 7$ matrix.
5. A linear transformation \( T : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) satisfies:

\[
T \left( \begin{bmatrix} 3 \\ 5 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} ; \\
T \left( \begin{bmatrix} 5 \\ 8 \end{bmatrix} \right) = \begin{bmatrix} -2 \\ 3 \end{bmatrix} .
\]

Find \( T \)'s standard matrix. For full credit, CHECK YOUR ANSWER. (A correct matrix without a check is worth 7 out of 10 points.) You won’t need fractions to solve this.

6. Use linear algebra to balance the following chemical equation (octane plus oxygen go to form carbon dioxide and water):

\[
C_8H_{18} + O_2 \rightarrow CO_2 + H_2O.
\]

You’ll need fractions here.

7. The Panem economy has two sectors, Snows and Mockingjays. The unit of currency is the gold piece. Snows sell \( \frac{4}{5} \) of their output to the Mockingjays and the rest to themselves. Mockingjays sell \( \frac{1}{10} \) of their output to the Snows and the rest to themselves. The total price of the Mockingjay output is 1000 gold pieces. What is the equilibrium price of the Snow output?

8. For what value(s) of \( h \) is the set of vectors \( \{(1,2,1), (-1,-1,1), (0,1,h)\} \) linearly independent in \( \mathbb{R}^3 \)?

9. Find a basis for the column space of

\[
\begin{bmatrix}
1 & 1 & 0 & 3 & 1 \\
-2 & -1 & -2 & -4 & -1 \\
1 & 2 & -2 & 6 & -1 \\
2 & 3 & -2 & 7 & 6
\end{bmatrix}.
\]

10. Find a basis for the null space of the matrix in problem #9.

Math 124 Quiz #3

Summer, 2014

The problems have equal credit. Show your work!

1. Solve the following system with Cramer’s Rule. For full credit, check your answer! (A correct answer without a check is worth 14 out of 20 points).

\[
2x_1 + 9x_2 = -3 \\
3x_1 + 5x_2 = 7.
\]
2. Suppose $A$ is a $6 \times 7$ matrix and $A(e_1 + 2e_2) = A(3e_1 - 5e_3) = 0$. Is $Ax = b$ solvable for all $b \in \mathbb{R}^6$, or do we need more information? Explain your answer.

3. Show that $\{-1+t+t^2, 1-t+t^2, 1+t-t^2\}$ forms a basis for $\mathcal{P}_2$, and find the coordinate vector of $2 + 3t + t^2$ with respect to it. You’ll need fractions for this.

4. Find the rank and nullity (dimension of the null space) of
\[
\begin{bmatrix}
1 & 1 & 3 & 1 & 6 & 1 \\
3 & 0 & 8 & 4 & 13 & 3 \\
4 & 1 & 11 & 5 & 19 & 4 \\
\end{bmatrix}.
\]

5. Suppose $B$ is a $6 \times 8$ matrix, $Bx = b$ is solvable for all $b \in \mathbb{R}^6$, and $B(e_1 + e_2) = B(e_2 + e_3) = 0$. Can $Be_3$ equal 0? Explain your answer.

Math 124 Quiz #4 Summer, 2014

The problems have equal credit. Show your work!

1. Find the complex eigenvalues and eigenvectors of the matrix:
\[
\begin{bmatrix}
3 & -2 \\
1 & 1 \\
\end{bmatrix}.
\]

2. The matrix $A$ given below has eigenvalues $-1$ and positive 2. Determine whether $A$ is diagonalizable. If it is diagonalizable, find an invertible matrix $P$ and a diagonal matrix $D$ such that $A = PDP^{-1}$.
\[
A = \begin{bmatrix}
-7 & -6 & 6 \\
9 & 8 & -9 \\
0 & 0 & -1 \\
\end{bmatrix}.
\]

3. Solve the initial value problem
\[
\begin{align*}
x'_1(t) &= -14x_1(t) + 30x_2(t) \\
x'_2(t) &= -6x_1(t) + 13x_2(t),
\end{align*}
\]
with initial condition $x_1(0) = x_2(0) = 1$.

4. Solve the dynamical system $x_{n+1} = Bx_n$, where
\[
B = \begin{bmatrix}
9/2 & -6 \\
2 & -5/2
\end{bmatrix}
\]
and
\[
x_0 = \begin{bmatrix}
1 \\
-1
\end{bmatrix}.
\]
The problems have equal credit. Show your work! Except where noted, your final numerical answers will not have fractions.

1. The matrix $A$ has eigenvalues 2 and 3. Determine whether $A$ is diagonalizable. If it is, find an invertible matrix $P$ and a diagonal matrix $D$ such that $A = PDP^{-1}$.

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix}.$$  

2. Find the vector $\hat{x} \in \mathbb{R}^2$ for which $\| b - B\hat{x}\|$ is as small as possible, where

$$B = \begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}$$

and

$$b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$  

Your answer will contain fractions.

3. Show that the vectors $v_1 = (2, 1, 1)$ and $v_2 = (-2, 3, 1)$ are orthogonal in $\mathbb{R}^3$, and find the projection of $c = (7, 0, 7)$ onto $Span\{v_1, v_2\}$.

4. Solve the system of differential equations

$$x'_1(t) = 5x_1(t) - 14x_2(t)$$

$$x'_2(t) = 3x_1(t) - 8x_2(t),$$

with initial condition $x_1(0) = 2$, $x_2(0) = 3$.

5. Solve the dynamical system $x_{k+1} = Rx_k$, where

$$R = \begin{bmatrix} 0 & -1 \\ 6 & 5 \end{bmatrix}$$

and

$$x_0 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}.$$  

6. Suppose $C$ is a $6 \times 9$ matrix, and $Cx = e_1 - 3e_3 + 7e_5$ is solvable with 3 free variables. Is $Cx = e_j$ solvable for all $1 \leq j \leq 6$, or do we need more information to tell? Explain!
7. Find the complex eigenvalues and eigenvectors of

\[
\begin{bmatrix}
-5 & 20 \\
-2 & 7
\end{bmatrix}.
\]

8. Prove that \(\{1 + 2t^2, 2 - t + t^2, -1 + 2t + t^2\}\) forms a basis for \(\mathcal{P}_2\), the polynomials of degree \(\leq 2\), and find the coordinate vector of \(-3 + 7t + 6t^2\) relative to it.

9. Find the rank and nullity of

\[
\begin{bmatrix}
2 & -1 & 4 & 3 \\
1 & 0 & 3 & 1 \\
1 & 1 & 5 & 0 \\
3 & 2 & 13 & 3
\end{bmatrix}.
\]

10. You are given data points \((-2, -1), (-1, -2), (0, 1),\) and \((1, 1)\) in the \(xy\)-plane. Find the \(y = mx + b\) equation of the line that best fits them, in the least-squares sense. Your final answer will have fractions.