ME 012 Engineering Dynamics

Lecture 8
Newton’s Laws of Motion, Equations of Motion, & Equations of Motion for a System of Particles
(Chapter 13, Sections 1, 2, and 3)

Thursday,
Feb. 07, 2013
Today’s Objectives:
Write the equation of motion for an accelerating body.
Draw the free-body and kinetic diagrams for an accelerating body.

In-Class Activities:
• Applications
• Newton’s Laws of Motion
• Newton’s Law of Gravitational Attraction
• Equation of Motion For A Particle or System of Particles
• Examples
APPLICATIONS

The motion of an object depends on the forces acting on it.

A parachutist relies on the atmospheric drag resistance force to limit his velocity.

Knowing the drag force, we can determine the acceleration or velocity of the parachutist at any point in time.

A freight elevator is lifted using a motor attached to a cable and pulley system as shown.

We determine the tension force in the cable required to lift the elevator at a given acceleration.
The motion of a particle is governed by Newton’s three laws of motion.

**First Law:** A particle originally at rest, or moving in a straight line at constant velocity, will remain in this state if the resultant force acting on the particle is zero.

**Second Law:** If the resultant force on the particle is not zero, the particle experiences an acceleration in the same direction as the resultant force. This acceleration has a magnitude proportional to the resultant force.

**Third Law:** Mutual forces of action and reaction between two particles are equal, opposite, and collinear.
The first and third laws were used in developing the concepts of statics. Newton’s second law forms the basis of the study of dynamics.

Mathematically, Newton’s second law of motion can be written

\[ F = ma \]

where \( F \) is the resultant unbalanced force acting on the particle, and \( a \) is the acceleration of the particle. The positive scalar \( m \) is called the mass of the particle.

Newton’s second law cannot be used when the particle’s speed approaches the speed of light, or if the size of the particle is extremely small (~ size of an atom).
NEWTON’S LAW OF GRAVITATIONAL ATTRACTION

Any two particles or bodies have a mutually attractive gravitational force acting between them. Newton postulated the law governing this gravitational force as

\[ F_1 = F_2 = F = G \frac{m_1 m_2}{r^2} \]

- \( F \) = force of attraction between the two bodies
- \( G \) = universal constant of gravitation
- \( m_1, m_2 \) = mass of each body
- \( r \) = distance between centers of the two bodies

When near the surface of the earth, the only gravitational force having any sizable magnitude is that between the earth and the body. This force is called the weight of the body.

\[ G = 6.67384 \times 10^{-11} \frac{m^3}{kg \cdot s^2} \]
MASS AND WEIGHT

It is important to understand the difference between the mass and weight of a body!

Mass is an absolute property of a body. It is independent of the gravitational field in which it is measured. The mass provides a measure of the resistance of a body to a change in velocity, as defined by Newton’s second law of motion \( m = F/a \).

The weight of a body is not absolute, since it depends on the gravitational field in which it is measured. Weight is defined as

\[ W = mg \]

where \( g \) is the acceleration due to gravity.
UNITS: SI SYSTEM VS. FPS SYSTEM

SI system: In the SI system of units, mass is a base unit and weight is a derived unit. Typically, mass is specified in kilograms (kg), and weight is calculated from $W = mg$. If the gravitational acceleration ($g$) is specified in units of m/s$^2$, then the weight is expressed in newtons (N). On the earth’s surface, $g$ can be taken as $g = 9.81$ m/s$^2$.

$$W \ (N) = m \ (kg) \ g \ (m/s^2) \Rightarrow N = kg \cdot m/s^2$$

FPS System: In the FPS system of units, weight is a base unit and mass is a derived unit. Weight is typically specified in pounds (lb), and mass is calculated from $m = W/g$. If $g$ is specified in units of ft/s$^2$, then the mass is expressed in slugs. On the earth’s surface, $g$ is approximately 32.2 ft/s$^2$:

$$m \ (slugs) = W \ (lb)/g \ (ft/s^2) \Rightarrow \text{slug} = \text{lb} \cdot s^2/ft$$
13.1 Newton’s Law of Motion

How Gravity is Determined

We normally deal with problems occurring on the surface of the Earth. With this in mind, the force between some object and the Earth itself resting on the Earth’s surface can be found through:

\[ F = G \frac{m_{object}m_{earth}}{d_{earth}^2} \]

By definition, the weight of the object, \( W \), is this represented force. We also know that weight is a mass times acceleration, or in this case, the mass of the object times the magnitude of gravity at the Earth’s surface:

\[ F = W = m_{object} \cdot g_{earth} = G \frac{m_{object}m_{earth}}{d_{earth}^2} \]

Cancelling out the mass of the object:

\[ g_{earth} = G \frac{m_{earth}}{d_{earth}^2} = 9.81 \text{ m/s}^2 = 32.2 \text{ ft/s}^2 \]

This gravitational force is not constant and is inversely proportional to the square of the distance between the two objects. Thus, the further you go from a body gravity is less... but still slightly present.
13.2 The Equation of Motion

The motion of a particle is governed by Newton’s second law, relating the unbalanced forces on a particle to its acceleration. If more than one force acts on the particle, the equation of motion can be written

\[ \sum F = F_R = ma \]

where \( F_R \) is the resultant force, which is a vector summation of all the forces.

To illustrate the equation, consider a particle acted on by two forces.

First, draw the particle’s free-body diagram, showing all forces acting on the particle. Next, draw the kinetic diagram, showing the inertial force \( ma \) acting in the same direction as the resultant force \( F_R \).
INERTIAL FRAME OF REFERENCE

This equation of motion is only valid if the acceleration is measured in a \textbf{Newtonian} or \textbf{inertial frame of reference}. What does this mean? Reference frame must be fixed or moving at constant velocity \emph{w.r.t.} object of interest so that the acceleration of the object of interest may be determined.

For problems concerned with motions at or near the earth’s surface, we typically assume our “inertial frame” to be \textbf{fixed to the earth}. We neglect any acceleration effects from the earth’s rotation.
The equation of motion can be extended to include systems of particles. This includes the motion of solids, liquids, or gas systems.

As in statics, there are internal forces and external forces acting on the system. What is the difference between them?

Using the definitions of \( m = \sum m_i \) as the total mass of all particles and \( \mathbf{a}_G \) as the acceleration of the center of mass \( G \) of the particles, then \( m\mathbf{a}_G = \sum m_i \mathbf{a}_i \).

The text shows the details, but for a system of particles: \( \sum \mathbf{F} = m\mathbf{a}_G \) where \( \sum \mathbf{F} \) is the sum of the external forces acting on the entire system.
KEY POINTS

1) **Newton’s second law** is a “Law of Nature”—experimentally proven and not the result of an analytical proof.

2) **Mass** (property of an object) is a measure of the **resistance to a change in velocity** of the object.

3) **Weight** (a force) depends on the **local gravitational field**. Calculating the weight of an object is an application of $F = ma$, i.e., $W = mg$.

4) **Unbalanced** forces cause the **acceleration** of objects. This condition is fundamental to all dynamics problems!
PROCEDURE FOR THE APPLICATION OF THE EQUATION OF MOTION

1) Select a convenient inertial coordinate system. Rectangular, normal/tangential, or cylindrical coordinates may be used.

2) Draw a free-body diagram showing all external forces applied to the particle. Resolve forces into their appropriate components. Use simple friction force relations \( F_f = \mu_k N \) and spring force relations \( F_s = ks \) when necessary.

3) Draw the kinetic diagram, showing the particle’s inertial force, \( ma \). Resolve this vector into its appropriate components.

4) Apply the equations of motion in their scalar component form and solve these equations for the unknowns.

5) It may be necessary to apply the proper kinematic relations to generate additional equations.
EXAMPLE 1

The driver attempts to tow the crate using a rope that has a tensile strength $T_{\text{max}} = 200 \text{ lb}$. If the crate is originally at rest and has weight $W = 500 \text{ lb}$, determine the greatest acceleration it can have if the coefficient of static friction between the crate and the road is $\mu_s = 0.4$ and the coefficient of kinetic friction is $\mu_s = 0.3$. 
EXAMPLE 1: Solution
EXAMPLE 2

The mine car of mass 400 kg is hoisted up the incline using the cable and motor $M$. For a short time, the force in the cable is $F = 3200t^2$. If the car has an initial velocity of 2 m/s when $t = 0$, determine its velocity and the distance it moves up the ramp when $t = 2$ s. The grade of the ramp, $c/d$, is $8/15$. 
EXAMPLE 2: Solution
EXAMPLE 3

Each of the two blocks has a mass $m$. The coefficient of kinetic friction at all surfaces of contact is $\mu$. If a horizontal force $P$ moves the bottom block, determine the acceleration of the bottom block in each case.
EXAMPLE 3: Solution