ME 012 Engineering Dynamics

Lecture 20

Absolute Motion Analysis and Relative Motion Analysis: Velocity
(Chapter 16, Sections 4 and 5)

Thursday,
Apr. 04, 2013
Chapter 16: Planar Kinematics of a Rigid Body

- 16.1 Rigid-Body Motion
- 16.2 Translation
- 16.3 Rotation About a Fixed Axis
- 16.4 Absolute Motion Analysis
- 16.5 Relative Motion Analysis: Velocity
- 16.6 Instantaneous Center of Zero Velocity
- 16.7 Relative Motion Analysis: Acceleration

Chapter 17: Planar Kinematics of a Rigid Body: Force and Acceleration

- 17.1 Moment of Inertia
- 17.2 Planar Kinetic Equations of Motion
- 17.3 Equations of Motion: Translation
- 17.4 Equations of Motion: Rotation About a Fixed Axis
- 17.5 Equations of Motion: General Plane Motion
TODAY’S OBJECTIVE

1. Determine the velocity and acceleration of a rigid body undergoing **general plane motion** using an absolute motion analysis.
2. Describe the velocity of a rigid body in terms of translation and rotation components.
3. Perform a relative-motion velocity analysis of a point on the body.

**In-Class Activities:**
- Applications
- General Plane Motion
- Translation and Rotation Components of Velocity
- Relative Velocity Analysis
- Problem Solving
APPLICATIONS

The position of the piston, $x$, can be defined as a function of the angular position of the crank, $\theta$. By differentiating $x$ with respect to time, the velocity of the piston can be related to the angular velocity, $\omega$, of the crank.

The stroke of the piston is defined as the total distance moved by the piston as the crank angle varies from 0 to 180°. How does the length of crank $AB$ affect the stroke?

The rolling of a cylinder is an example of general plane motion.

During this motion, the cylinder rotates clockwise while it translates to the right.

The position of the center, $G$, is related to the angular position, $\theta$, by, $s_G = r\theta$, if the cylinder rolls without slipping.

You relate the translational velocity of $G$ and the angular velocity of the cylinder.
16.4 Absolute Motion Analysis

**Absolute motion analysis** (also called the parametric method) is used to study planar motion.

- Recall, a body subjected to *general plane motion* undergoes a simultaneous **translation** and **rotation**
- This method relates the position of a point, \( P \), on a rigid body undergoing rectilinear motion to the angular position, \( \theta \) (parameter), of a line contained in the body.
- Often this line is a link in a machine.
- Once a relationship in the form of \( s_P = f(\theta) \) is established, the velocity and acceleration of point \( P \) are obtained in terms of the angular velocity, \( \omega \), and angular acceleration, \( \alpha \), of the rigid body by taking the **first and second time derivatives** of the position function.

\[
\begin{align*}
  s_P &= f(r, \theta) \\
  v_P &= \frac{d[f(\theta)]}{dt} = f(r, \theta, \omega) \\
  a_P &= \frac{d^2[f(\theta)]}{dt^2} = f(r, \theta, \omega, \alpha)
\end{align*}
\]

- Usually the **chain rule** must be used when taking the derivatives of the position coordinate equation.
Using trigonometry, a relation between the rotational motion of OA and rectilinear translation of rod R (measured from fixed point O):

\[ x = 2r \cos \theta \]

Using the chain rule:

\[ \frac{dx}{dt} = -2r (\sin \theta) \frac{d\theta}{dt} \quad \Rightarrow \quad v = -2r \omega (\sin \theta) \]

\[ \frac{dv}{dt} = -2r \left( \frac{d\omega}{dt} \right) (\sin \theta) - 2r \omega (\cos \theta) \frac{d\theta}{dt} \quad \Rightarrow \quad a = -2r [\alpha (\sin \theta) + \omega^2 (\cos \theta)] \]
16.5 Relative Motion Analysis: Velocity

When a body is subjected to general plane motion, it undergoes a combination of translation and rotation.

- The position vector $\mathbf{r}_A$ specifies the location of “base point” $A$.
- This point generally has a known motion.
- The position vector of point $B$ ($\mathbf{r}_B$) can be related to a selected “base-point” $A$ via:

\[
\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}
\]
When a body is subjected to general plane motion, it undergoes a combination of **translation** and **rotation**.

**DISPLACEMENT**

- During time duration $dt$, points $A$ and $B$ undergo displacements $dr_A$ and $dr_B$.
- Allow base point $A$ to translate to new origin.
- The solid body is rotated by $d\theta$.
- Due to this rotation, $dr_{B/A} = r_{B/A}d\theta$.
- The displacement of $B$ is then:

$$dr_B = dr_A + dr_{B/A}$$

Due to translation and rotation

Due to translation of $A$

Due to rotation about $A$
16.5 Relative Motion Analysis: Velocity

The velocity at $B$ is given as:

$$\frac{d\mathbf{r}_B}{dt} = \frac{d\mathbf{r}_A}{dt} + \frac{d\mathbf{r}_{B/A}}{dt}$$

OR

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

Since the body is taken as rotating about $A$,

$$\mathbf{v}_{B/A} = \frac{d\mathbf{r}_{B/A}}{dt} = \omega \times \mathbf{r}_{B/A}$$

Here $\omega$ will only have a $\mathbf{k}$ component since the axis of rotation is perpendicular to the plane of translation.

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$$
16.5 Relative Motion Analysis: Velocity

When using the relative velocity equation, points $A$ and $B$ should generally be points on the body with a known motion. Often these points are pin connections in linkages.

Here both points $A$ and $B$ have circular motion since the disk and link $BC$ move in circular paths. The directions of $\mathbf{v}_A$ and $\mathbf{v}_B$ are known since they are always tangent to the circular path of motion.

\[ \mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A} \]
When a wheel rolls without slipping, point $A$ is often selected to be at the point of contact with the ground. Since there is no slipping, point $A$ has zero velocity.

Furthermore, point $B$ at the center of the wheel moves along a horizontal path. Thus, $\mathbf{v}_B$ has a known direction, e.g., parallel to the surface.
The **relative velocity equation** can be applied using either a Cartesian vector analysis or by writing scalar $x$ and $y$ component equations directly.

**PROCEDURE FOR SCALAR ANALYSIS**

1. Establish the fixed $x$-$y$ coordinate directions and draw a **kinematic diagram** for the body. Then establish the magnitude and direction of the relative velocity vector $\mathbf{v}_{B/A}$.

2. Write the equation $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$ and by using the kinematic diagram, underneath each term represent the vectors graphically by showing their **magnitudes and directions**.

3. Write the scalar equations from the $x$ and $y$ components of these graphical representations of the vectors. Solve for the unknowns.
PROCEDURE FOR VECTOR ANALYSIS

1. Establish the fixed $x$-$y$ coordinate directions and draw the **kinematic diagram** of the body, showing the vectors $\mathbf{v}_A$, $\mathbf{v}_B$, $\mathbf{r}_{B/A}$ and $\omega$. If the magnitudes are unknown, the sense of direction may be assumed.

2. Express the vectors in **Cartesian vector form** and substitute into $\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$. Evaluate the cross product and equate respective $i$ and $j$ components to obtain **two** scalar equations.

3. If the solution yields a **negative** answer, the sense of direction of the vector is **opposite** to that assumed.
EXAMPLE 1

The bar of length $L = 5$ ft is confined to move along a vertical and inclined plane ($\phi = 30^\circ$). The velocity of the roller at $A$ is $v_A = 6$ ft/s downward when $\theta = 45^\circ$. Determine the bar's angular velocity and the velocity of roller $B$ at this instant.
EXAMPLE 2

A wheel ($r_B = 150$ mm) rotating with an angular velocity $\omega_B = 8$ rad/s is connected to collar $A$ by a rigid rod ($r_A = 500$ mm). Determine the velocity of the collar $A$ at the instant $\theta = 30^\circ$ and $\phi = 60^\circ$. 
EXAMPLE 3

The pinion gear \((r = 0.3 \text{ ft})\) rolls on the gear racks. Rack \(B\) is moving to the right at speed \(v_B = 8 \text{ ft/s}\) and rack \(C\) is moving to the left at speed \(v_C = 4 \text{ ft/s}\). Determine the angular velocity of the pinion gear and the velocity of its center \(A\).
EXAMPLE 3: Solution