ME 012 Engineering Dynamics

Lecture 13
Conservative Forces and Potential Energy and Conservation of Energy
(Chapter 14, Sections 5 and 6)

Thursday,
Feb. 28, 2013
CONSERVATIVE FORCES AND POTENTIAL ENERGY AND CONSERVATION OF ENERGY

Today’s Objectives:
• Understand the concept of conservative forces and determine the potential energy of such forces.
• Apply the principle of conservation of energy.

In-Class Activities:
• Applications
• Conservative Force
• Potential Energy
• Conservation of Energy
• Problem Solving
APPLICATIONS

The weight of the sacks resting on this platform causes potential energy to be stored in the supporting springs.

If the sacks weight is known and the equivalent spring constant $k$ is given, then the energy stored in the springs can be calculated.

A gantry structure is used to test the response of an aircraft during a crash. The plane is supported by a rigid cable and allowed to swing as a pendulum.

By raising the plane a certain elevation energy is stored in potential form. This potential energy is transformed into kinetic energy as the plane swings to impact. This elevation can be tailored to meet crash specifications for kinetic energy upon impact.
CONSERVATIVE FORCE

A force \( F \) is said to be conservative if the work done is independent of the path followed by the force acting on a particle as it moves from A to B. In other words, the work done by the force \( F \) in a closed path (i.e., from A to B and then back to A) equals zero:

\[
U_{A-B} = -U_{B-A}
\]

This means the work is conserved.

A conservative force depends only on the position of the particle, and is independent of its velocity or acceleration.

Examples of **conservative “saving” forces**:  
• Weight of a body: depends on the vertical displacement of the weight  
• Force developed by a spring: depends only on the springs elongation or compression

Example of **non-conservative “wasting” force**:  
• Friction: work is dissipated into the body in the form of heat

\[
dU = F \cdot dr
\]

\( F \) is a conservative force if its integral over any closed path is zero:

\[
\int F \cdot dr = 0
\]
ENERGY

Energy is defined as the capacity to do work \((U)\). Recall, if a particle is initially at rest then the work that must be done to bring the particle to speed \(v\) is:

\[
T_1^0 + \Sigma U_{1-2} = T_2 = \frac{1}{2}mv^2
\]

**Kinetic Energy:** is a measure of the particle’s capacity to do work associated with the motion of the particle

**Potential Energy:** is a measure of the amount of work a conservative force will do when it moves from a given position to the datum.
Potential energy is a measure of the amount of work a conservative force will do when it changes position.

In general, for any conservative force system, we can define the potential function \( V \) as a function of position. The work done by conservative forces as the particle moves equals the change in the value of the potential function (essentially the sum of the potential energies).

It is important to become familiar with the two types of potential energy and how to calculate their magnitudes:

- Gravitational Potential Energy
- Elastic Potential Energy (i.e. spring stored energy)
POTENTIAL ENERGY DUE TO GRAVITY

The potential function (formula) for a gravitational force, e.g., weight \((W = mg)\), is the force multiplied by its elevation from a datum. The datum can be defined at any convenient location.

\[
V_g = \pm Wy
\]

\(V_g\) is positive if \(y\) is above the datum and negative if \(y\) is below the datum. Remember, YOU get to set the datum.
Recall that the force of an elastic spring is $F = ks$. It is important to realize that the potential energy of a spring, while it looks similar, is a different formula.

$V_e$ (where ‘e’ denotes an elastic spring) has the distance $s$ raised to a power (the result of an integration) or:

$$V_e = \frac{1}{2}ks^2$$

Notice that the potential function $V_e$ always yields positive energy.
POTENTIAL FUNCTION

A more rigorous definition of a conservative force makes use of a potential function \( V \) and partial differential calculus, as explained in the texts. However, even without the use of these mathematical relationships, much can be understood and accomplished.

The “conservative” potential energy of a particle/system is typically written using the potential function \( V \). If a particle is subjected to both gravitational and elastic forms of potential energy, the particle’s energy can be expressed as the potential function in its algebraic form:

\[
V = V_{\text{gravity}} + V_{\text{springs}}
\]
When a particle is acted upon by a system of conservative forces, the work done by these forces is conserved and the sum of kinetic energy and potential energy remains constant. In other words, as the particle moves, kinetic energy is converted to potential energy and vice versa. This principle is called the principle of conservation of energy and is expressed as:

\[ T_1 + V_1 = T_2 + V_2 = \text{constant} \]

\( T_1 \) stands for the kinetic energy at state 1 and \( V_1 \) is the potential energy function for state 1. \( T_2 \) and \( V_2 \) represent these energy states at state 2. Recall, the kinetic energy is defined as:

\[ T = \frac{1}{2} m v^2 \]

If a system of particles is subjected only to conservative forces:

\[ \Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2 = \text{constant} \]
14.6 Conservation of Energy

Consider a ball of weight $W$ dropped from a height $h$ \((1)\)

Right before it is dropped \((1)\) the potential energy is at a maximum and the kinetic energy is 0 thus the total energy is:

$$E_1 = V_1 + T_1 = Wh + 0 \implies E_1 = Wh$$

When the ball has fallen a distance of $h/2$ \((2)\) the speed can be determined by constant acceleration relations:

$$v^2 = v_0^2 + 2a_c(y - y_0)$$
$$= 2g \frac{h}{2} \implies v = \sqrt{gh}$$

$$E_2 = V_2 + T_2 = W \frac{h}{2} + \frac{1}{2}mv^2$$
$$= W \frac{h}{2} + \frac{1}{2}mgh \implies E_2 = Wh$$

When the ball strikes the ground \((3)\) the ball has travelled a distance $h$ and its speed can be determined by:

$$v^2 = v_0^2 + 2a_c(y - y_0) = 2gh \implies v = \sqrt{2gh}$$

$$E_3 = V_3 + T_3 = 0 + \frac{1}{2}mv^2$$
$$= 0 + mgh \implies E_3 = Wh$$
14.5 & 14.6 Procedure for Analysis

POTENTIAL ENERGY

• Draw two diagrams showing particle located in initial and final points along path
• If vertical displacement present, establish fixed horizontal datum from which to measure particle’s gravitational potential energy $V_g$
• Elevation of particle from the datum, $y$, and stretch or compression, $s$, of any connecting springs can be determined from geometry associated with two diagrams
• Gravitational Potential: $V_g = \mathcal{W}y$ where $y$ is positive upward from datum and negative downward from datum
• Spring Potential: $V_e = \frac{1}{2}ks^2$ which is always positive

CONSERVATION OF ENERGY

• Apply the relation: $T_1 + V_1 = T_2 + V_2 = \text{constant}$
• Don’t forget that the velocity for the relation $T = \frac{1}{2}mv^2$ must be measured from an inertial reference frame (where $a = 0$)
EXAMPLE 1

The bob of the pendulum has a mass $M = 0.2$ kg and is released from rest when it is in the horizontal position shown. Determine its speed and the tension in the cord at the instant the bob passes through its lowest position if the cable length is $r = 0.75$ m.
EXAMPLE 1: Solution
EXAMPLE 2

The girl and bicycle weigh 125 lbs. She moves from point $A$ to $B$. Determine the velocity and the normal force at $B$ if the velocity at $A$ is 10 ft/s and she stops pedaling at $A$. 

![Diagram showing a girl and bicycle on a slope with points A and B, and a velocity of 10 ft/s at A.](image)
EXAMPLE 2: Solution
EXAMPLE 3

The collar of weight $W = 5\ \text{lb}$ is released from rest at $A$ and travels along the frictionless guide. Determine the speed of the collar just before it strikes the stop at $B$. Assume the spring has an unstretched length $L = 12\ \text{in}$, height to point $C$ is $h = 10\ \text{in}$, and the spring constant is $k = 2\ \text{lb/in}$. 
EXAMPLE 3: Solution