ME 012 Engineering Dynamics

Lecture 12
Power and Efficiency
(Chapter 14, Sections 4)

Tuesday,
Feb. 26, 2013
The initial sum of kinetic energy of a system plus the sum of work done on the system is equal to the final kinetic energy of the system.

\[ \sum T_1 + \sum U_{1-2} = \sum T_2 \]

\[ T_1 = \frac{1}{2} m(v_1)^2 \]
\[ T_2 = \frac{1}{2} m(v_2)^2 \]

**WEIGHT**

\[ (U_{1-2})_W = -W \Delta y \]

- If particle object is being pushed upward, work from weight is negative.
- If particle object is falling, work from weight is positive.

**FRICITION**

\[ (U_{1-2})_F = \mu_k N s \]

**SPRING FORCE**

\[ (U_{1-2})_S = \int_{s_1}^{s_2} F_s ds \]

**BE CAREFUL:**

Spring might not be a linear relation between force and elongation-compression!!
14.1 to 14.3: Work and Energy Quick Review

(A) The spring here is in a relaxed state

(B) However, in this system there exists a stopper (indicated in red) which now represents the springs initial state as pre-compressed to ‘d’

(C) Pulling the spring back a distance ‘x’ from state (B) represents the amount of spring force work will be done on the ball (going from ‘d’ to ‘d+x’)

(D) In a frictionless environment, the weight of the ball is the only force opposing the spring work. It is essentially a mechanism that is storing potential energy.

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- Last example we wanted to know when the ball began to fall off the track. Final state K.E. required calculation of velocity through normal force balance where N=0.
- What would the final state K.E. be if you wanted to calculate the distance required to pull the spring back to have the ball just reach a height of \( h \)?
Today’s Objectives:
- Determine the power generated by a machine, engine, or motor.
- Calculate the mechanical efficiency of a machine.

In-Class Activities:
- Review Work and Energy
- Applications
- Define Power
- Define Efficiency
- Problem Solving
14.4 Power and Efficiency

APPLICATIONS

Engines and motors are often rated in terms of their power output. The power requirements of the motor lifting this elevator depend on the vertical force $F$ that acts on the elevator, causing it to move upwards.

Given the desired lift velocity for the elevator, we can determine the power requirement of the motor.

The speed at which a vehicle can climb a hill depends in part on the power output of the engine and the angle of inclination of the hill.

For a given angle, we can determine the speed of this jeep, knowing the power transmitted by the engine to the wheels?
**POWER**

Power is defined as the amount of work performed per unit of time:

\[ \text{Power} = \frac{\text{Work}}{\text{Time}} \]

If a machine or engine performs a certain amount of work, \( dU \), within a given time interval, \( dt \), the power generated can be calculated as:

\[ P = \frac{dU}{dt} \]

Since the work can be expressed as: \( dU = F \cdot dr \)

the power can be written as:

\[ P = \frac{dU}{dt} = \frac{F \cdot dr}{dt} = F \cdot \frac{dr}{dt} = F \cdot v \]

Thus, power is a scalar defined as the product of the force and velocity components acting in the same direction.
14.4 Power and Efficiency

POWER

Using scalar notation, power can be written

\[ P = \mathbf{F} \cdot \mathbf{v} = Fv \cos \theta \]

where \( \theta \) is the angle between the force and velocity vectors.

So if the velocity of a body acted on by a force \( \mathbf{F} \) is known, the power can be determined by calculating the dot product or by multiplying force and velocity components.

POWER UNITS

The unit of power in the SI system is the watt (W) where:

\[ 1 \text{ W} = 1 \text{ J/s} = 1 (\text{N} \cdot \text{m})/\text{s} \]

In the FPS system, power is usually expressed in units of horsepower (hp) where:

\[ 1 \text{ hp} = 550 (\text{ft} \cdot \text{lb})/\text{s}. \]

Conversion between SI and FPS units of power can be done with the simple relation:

\[ 1 \text{ hp} = 746 \text{ W} \]
EFFICIENCY

The mechanical efficiency of a machine is the ratio of the useful power produced (output power) to the power supplied to the machine (input power) or:

\[ \varepsilon = \frac{\text{power output}}{\text{power input}} \]

If energy input and removal occur at the same time, efficiency may also be expressed in terms of the ratio of output energy to input energy or:

\[ \varepsilon = \frac{\text{energy output}}{\text{energy input}} \]

- Machines will always have frictional forces.
- Since frictional forces dissipate energy, additional power will be required to overcome these forces.
- Consequently, the efficiency of a machine is always less than 1.
PROCEDURE FOR ANALYSIS

• Find the **resultant external force** acting on the body causing its motion. It may be necessary to draw a free-body diagram.

• Determine the **velocity** of the **point** on the body **at which the force is applied**. Energy methods or the equation of motion and appropriate kinematic relations, may be necessary.

• Multiply the **force magnitude** by the component of **velocity** acting in the **direction** of \( F \) to determine the power supplied to the body \( (P = Fv \cos \theta) \).

• In some cases, **power** may be found by calculating the **work done per unit of time** \( (P = \frac{dU}{dt}) \).

• If the **mechanical efficiency** of a machine is known, either the power input or output can be determined.
EXAMPLE 1

A linear spring having a stiffness $k = 5$ kN/m is compressed a distance $\delta = 400$ mm. The stored energy in the spring is used to drive a machine which requires power $P = 90$ W. Determine how long the spring can supply energy at the required rate.
EXAMPLE 1: Solution
EXAMPLE 2

A truck has a weight $W = 25000$ lbf and an engine which transmits a power $P = 350$ hp to all the wheels. Assuming that the wheels do not slip on the ground, determine the angle $\theta$ of the largest incline the truck can climb at a constant speed $v = 50$ ft/sec.
EXAMPLE 2: Solution
EXAMPLE 3

A 50-lb load (B) is hoisted by the pulley system and motor M. The motor has an efficiency of 0.76 and exerts a constant force of 30 lb on the cable. Neglect the mass of the pulleys and cable. The power supplied to the motor when the load has been hoisted 10 ft. The block started from rest.
EXAMPLE 3: Solution