(1) What does \((2 - \zeta)^{-1}(x, y)\) count in an arbitrary locally finite poset?

(2) Simplify the formula \(\sum_{d|m} \sum_{k|d} \mu(k)g(d/k)\).

(3) Prove that if \(f(m)\) and \(g(m)\) are multiplicative functions, then so is \(h(m) = \sum_{d|m} f(d)g(m/d)\).

(4) (a) Prove that \(\sum_{d|n} \varphi(d) = n\) (\(\varphi\) is Euler’s totient function).
   
   (b) Apply Möbius inversion to express \(\varphi(n)\) as a sum over the divisors of \(n\).

(5) (Wilf, problem 2.23) Let \(\{B_n\}\) be the sequence of Bernoulli numbers defined by the exponential generating function \(x/(e^x - 1)\). Let \(m\) be a positive integer. By considering the generating function
   \[
x \frac{e^{mx} - 1}{e^x - 1}
   \]
in two ways, find an evaluation of the sum of the \(r\)th powers of the first \(N\) positive integers as a polynomial of degree \(r + 1\) in \(N\), whose coefficients are given quite explicitly in terms of the Bernoulli numbers.