(1) Prove that \( \frac{1}{\sqrt{1-4x}} = \sum_{k \geq 0} \binom{2k}{k} x^k \).

(2) Give a combinatorial justification for the formula
\[
\begin{bmatrix} n \\ k \end{bmatrix} = \sum_{a_1 + a_2 + \ldots + a_k = n-k; a_i \geq 0} 1^{a_1} 2^{a_2} \ldots k^{a_k}.
\]

(3) Index the Fibonacci numbers as Wilf does: \( F_0 = 0, \ F_1 = 1, \ F_{n+1} = F_n + F_{n-1} \) for \( n \geq 1 \). Fix \( n > 0 \). Find a closed formula for
\[
\sum_{m > 0} \sum_{k_1 + k_2 + \ldots + k_m = n} F_{k_1} F_{k_2} \ldots F_{k_m}.
\]

(4) Find a generating function \( G(x) \) such that
\[
[x^n] G(x) = \sum_k \binom{r}{k} \binom{r}{n-2k}.
\]

**Suggested (but non-graded) problems.** Answers are in the back of the textbook: Chapter 1) 2,4,5