A few solutions:

(1) \( \sigma = [4, 2, 6, 3, 1, 5] = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 2 & 6 & 3 & 1 & 5 \end{bmatrix} = (1, 4, 3, 6, 5)(2). \)

(2) First we write in cycle notation: \((1)(2, 9, 6, 7, 5)(3, 4, 8). \) Then rewrite by splitting each cycle into a product of 2-cycles: \((2, 9)(9, 6)(6, 7)(7, 5)(3, 4)(4, 8). \)

(3) Start with 1. Reading the cycles from right to left, so where it gets sent to (within each cycle read left to right). The third cycle from the right sends 1 \( \mapsto \) 5. The next one to the left does not move 5. The leftmost sends 5 \( \mapsto \) 3. So we start out writing “(1, 3”. Now we see what happens to 3 to decide whether to close this cycle or append another number. We see that 3 \( \mapsto \) 4 \( \mapsto \) 3 \( \mapsto \) 8 \( \mapsto \) 8. So we continue with “(1, 3, 8”. Continuing in this manner we finish off the first cycle as (1, 3, 8, 2). Start the next cycle “(4” since 4 is the smallest number we haven’t dealt with yet. Etc. You should end up with (1, 3, 8, 2)(4)(5, 6, 7).

(4) \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 4 & 5 & 2 & 1 \end{bmatrix}.

(5) No. The elements of \( S_6 \) are bijections from \( \{1, 2, 3, 4, 5, 6\} \) to itself. \( \{1, 4, 3, 2, 5\} \) is a set, so it’s not even an object of the right type.