Additional Examples of Chapter 5: 
Finite-Length Discrete Transforms

Example E5.1: Consider the following length-8 sequences defined for \(0 \leq n \leq 7\):

(a) \(\{x_1[n]\} = \{1 1 1 0 0 0 1 1\}\),  (b) \(\{x_2[n]\} = \{1 1 0 0 0 0 -1 -1\}\),
(c) \(\{x_3[n]\} = \{0 1 1 0 0 0 -1 -1\}\),  (d) \(\{x_4[n]\} = \{0 1 1 0 0 0 1 1\}\).

**Answer:** (a) \(x_1[-n] = 1110 0011\) = \(x_1[n]\). Thus, \(x_1[n]\) is a periodic even sequence, and hence it has a real-valued 8-point DFT.

(b) \(x_2[-n] = 1 -1 -1 0 0 0 0 1\). Thus, \(x_2[n]\) is neither a periodic even nor a periodic odd sequence. Hence, its 8-point DFT is a complex sequence.

(c) \(x_3[-n] = 0 -1 -1 0 0 0 1 1\) = \(-x_3[n]\). Thus, \(x_3[n]\) is a periodic odd sequence, and hence it has an imaginary-valued 8-point DFT.

(d) \(x_4[-n] = 0 1 1 0 0 0 1 1\) = \(x_4[n]\). Thus, \(x_4[n]\) is a periodic even sequence, and hence it has a real-valued 8-point DFT.

Example E5.2: Let \(G[k]\) and \(H[k]\) denote the 7-point DFTs of two length-7 sequences, \(g[n]\) and \(h[n]\), \(0 \leq n \leq 6\), respectively. If

\[
\begin{align*}
G[k] &= \{1 + j2, -2 + j3, -1 - j2, 0, 8 + j4, -3 + j, 2 + j5\} \\
h[n] &= g[\langle n - 3 \rangle_7],
\end{align*}
\]

determine \(H[k]\) without computing the IDFT \(g[n]\).

**Answer:**

\[
H[k] = \text{DFT}\{h[n]\} = \text{DFT}\{g[\langle n - 3 \rangle_7]\} = W_7^{3k} G[k] = e^{-j\frac{6\pi k}{7}} G[k]
\]

\[
= \begin{bmatrix}
1 + j2, & e^{-j\frac{6\pi}{7}} (-2 + j3), & e^{-j\frac{12\pi}{7}} (-1 - j2), & 0, & e^{-j\frac{24\pi}{7}} (8 + j4), & e^{-j\frac{30\pi}{7}} (-3 + j2), & e^{-j\frac{36\pi}{7}} (2 + j5)
\end{bmatrix}
\]

Example E5.3: Let \(G[k]\) and \(H[k]\) denote the 7-point DFTs of two length-7 sequences, \(g[n]\) and \(h[n]\), \(0 \leq n \leq 6\), respectively. If \(g[n] = \{-3.1, 2.4, 4.5, -6, 1, -3, 7\}\) and \(G[k] = H[\langle k - 4 \rangle_7]\), determine \(h[n]\) without computing the DFT \(g[n]\).

**Answer:**

\[
h[n] = \text{IDFT}\{H[k]\} = \text{IDFT}\{G[\langle k - 4 \rangle_7]\} = W_7^{-4n} g[n] = e^{j\frac{8\pi n}{7}} g[n]
\]

\[
= \begin{bmatrix}
-3.1, & 2.4 e^{j8\pi/7}, & 4.5 e^{j16\pi/7}, & -6 e^{j24\pi/7}, & e^{j32\pi/7}, & -3 e^{j40\pi/7}, & 7 e^{j42\pi/7}
\end{bmatrix}
\]

Example E5.4: Let \(X[k]\), \(0 \leq k \leq 13\), be a 14-point DFT of a length-14 real sequence \(x[n]\). The first 8 samples are given by \(X[0] = 12, X[1] = -1+j3, X[2] = 3+j4, X[3] = 1-j5, X[4] = -2+j2, X[5] = 6+j3, X[6] = -2-j3, X[7] = 10\). Determine the remaining samples of \(X[k]\). Evaluate the following functions of \(x[n]\) without computing the IDFT of \(X[k]\):
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(a) $x[0]$, (b) $x[7]$, (c) $\sum_{n=0}^{13} x[n]$, (d) $\sum_{n=0}^{13} e^{j(4\pi n/7)} x[n]$, and (e) $\sum_{n=0}^{13} |x[n]|^2$.

**Answer:**


(a) $x[0] = \frac{1}{14} \sum_{k=0}^{13} X[k] = \frac{32}{14} = 2.2857$,

(b) $x[7] = \frac{1}{14} \sum_{k=0}^{13} (-1)^k X[k] = -\frac{12}{14} = -0.8571$,

(c) $\sum_{n=0}^{13} x[n] = X[0] = 12$,

(d) Let $g[n] = e^{j(4\pi n/7)} x[n] = W_{14}^{-4n} x[n]$. Then $DFT\{g[n]\} = DFT\{W_{14}^{-4n} x[n]\} = X[k - 4]_{14}$


Thus, $\sum_{n=0}^{13} g[n] = \sum_{n=0}^{13} e^{j(4\pi n/7)} x[n] = X[10] = -2 + j2$,

(e) Using Parseval's relation, $\sum_{n=0}^{13} |x[n]|^2 = \frac{1}{14} \sum_{k=0}^{13} |X[k]|^2 = \frac{498}{14} = 35.5714$.

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**Example E5.5:** Consider the sequence $x[n]$ defined for $0 \leq n \leq 11$,

$\{x[n]\} = \{3, -1, 2, 4, -3, -2, 0, 1, -4, 6, 2, 5\}$,

with a 12-point DFT given by $X[k]$, $0 \leq k \leq 11$. Evaluate the following functions of $X[k]$ without computing the DFT:

(a) $X[0]$, (b) $X[6]$, (c) $\sum_{k=0}^{11} X[k]$, (d) $\sum_{k=0}^{11} e^{-j(2\pi k/3)} X[k]$, and (e) $\sum_{k=0}^{11} |X[k]|^2$.

**Answer:**

- **X[0] = $\sum_{n=0}^{11} x[n] = 13$, X[6] = $\sum_{n=0}^{11} (-1)^n x[n] = -13$,**
- **$\sum_{k=0}^{11} X[k] = 12 \cdot x[0] = 36$, (d) The inverse DFT of $e^{-j(4\pi k/6)} X[k]$ is $x[\langle n - 4 \rangle_{12}]$. Thus,**

$\sum_{k=0}^{11} e^{-j(4\pi k/6)} X[k] = 12 \cdot x[\langle n - 4 \rangle_{12}] = 12 \cdot x[8] = -48$.

- **From Parseval's relation, $\sum_{k=0}^{11} |X[k]|^2 = 12 \cdot \sum_{n=0}^{11} |x[n]|^2 = 1500$.**
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**Answer:** Since \(x[n]\) is a real sequence, its DFT satisfies \(X[k] = X^*[(-k)_N]\) where \(N = 11\) in this case. Therefore, \(X[1] = X^*[(-1)_{11}] = X^*[10] = \sqrt{3} + j2,\)

---

Example E5.7: The following 6 samples of the 11-point DFT \(X[k],\) \(0 \leq k \leq 10,\) are given: \(X[0] = 12, X[2] = -3.2 - j2, X[3] = 5.3 - j4.1, X[5] = 6.5 + j9, X[7] = -4.1 + j0.2,\) and \(X[10] = -3.1 + j5.2.\) Determine the remaining 5 samples.

**Answer:** The \(N\)-point DFT \(X[k]\) of a length-\(N\) real sequence \(x[n]\) satisfy \(X[k] = X^*[(-k)_N].\) Here \(N = 11.\) Hence, the remaining 5 samples are \(X[1] = X^*[(-1)_{11}] = X^*[10] = -3.1 - j5.2,\)

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Example E5.8: A length-10 sequence \(x[n],\) \(0 \leq n \leq 9,\) has a real-valued 10-point DFT \(X[k],\)
\(0 \leq k \leq 9.\) The first 6 samples of \(x[n]\) are given by: \(x[0] = 2.5, x[1] = 0.7 - j0.08, x[2] = -3.25 + j1.12, x[3] = -2.1 + j4.6, x[4] = 2.87 + j2,\) and \(x[5] = 5.\) Determine the remaining 4 samples.

**Answer:** A length-\(N\) periodic even sequence \(x[n]\) satisfying \(x[n] = x^*[(-n)_N]\) has a real-valued \(N\)-point DFT \(X[k].\) Here \(N = 10.\) Hence, the remaining 4 samples of \(x[n]\) are given by \(x[6] = x^*[(-6)_{10}] = x^*[4] = 2.87 - j2,\)
\(x[7] = x^*[(-7)_{10}] = x^*[3] = -2.1 - j4.6,\)
\(x[8] = x^*[(-8)_{10}] = x^*[2] = -3.25 - j1.12,\) and \(x[9] = x^*[(-9)_{10}] = x^*[1] = 0.7 + j0.08.\)

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Example E5.9: The 8-point DFT of a length-8 complex-valued sequence \(v[n] = x[n] + jy[n]\) is given by
\[
\{V[k]\} = \{-2 + j3, \quad 1 + j5, \quad -4 + j7, \quad 2 + j6, \quad -1 - j3, \quad 4 - j, \quad 3 + j8, \quad j6.\}
\]
Without computing the IDFT of \(V[k],\) determine the 8-point DFTs \(X[k]\) and \(Y[k]\) of the real sequences \(x[n]\) and \(y[n],\) respectively.

**Answer:** \(v[n] = x[n] + jy[n].\) Hence, \(X[k] = \frac{1}{2} \{V[k] + V^*[(-k)_8]\}\) is the 8-point DFT of \(x[n],\)
and \(Y[k] = \frac{1}{2j} \{V[k] - V^*[(-k)_8]\}\) is the 8-point DFT of \(y[n].\) Now,
\[V^*[(-k)_8] = [-2 - j3, \quad -j6, \quad 3 - j8, \quad 4 + j, \quad -1 + j3, \quad 2 - j6, \quad -4 - j7, \quad 1 - j5].\]
Therefore,
\[X[k] = [-0.2, \quad 0.5 - j0.5, \quad -0.5 - j0.5, \quad 3 + j3.5, \quad -1, \quad 3 - j3.5, \quad -0.5 + j0.5, \quad 0.5 + j0.5],\]
\[Y[k] = [3, \quad 5.5 - j0.5, \quad 7.5 + j3.5, \quad 2.5 + j - 3, \quad 2.5 - j, \quad 7.5 - j3.5, \quad 5.5 + j0.5].\]
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**Example E5.10:** Determine the 4-point DFTs of the following length-4 sequences, \(0 \leq n \leq 3\), defined for \(g[n]\) by computing a single DFT: \(g[n] = \{-2, \ 1, \ -3, \ 4\}\), \(h[n] = \{1, \ 2, \ -3, \ 2\}\).

**Answer:**
\[
V[0] = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -2 + j \ 1 + j2 \ -3 - j3 \ 4 + j2 \end{bmatrix} = j2 \] 
\[
V[1] = \begin{bmatrix} 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 1 + j2 \ -3 - j3 \end{bmatrix} = 1 + j7 \] 
\[
V[2] = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} j2 \end{bmatrix} = 1 + j \] 
\[
V[3] = \begin{bmatrix} 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 + j2 \end{bmatrix} = 1 + j \] 

Thus, \(V^{*}[\langle -k \rangle_4] = [-j2, \ 1 - j, \ -10 + j6, \ 1 - j7]\).

Therefore, \(G[k] = \frac{1}{2} \{V[k] + V^{*}[\langle -k \rangle_4]\} = [0, \ 1 + j3, \ -10, \ 1 - j3]\) and \(H[k] = \frac{1}{2j} \{V[k] - V^{*}[\langle -k \rangle_4]\} = [2, \ 4, \ -6, \ 4]\).