NAME

Do all problems

Please write clearly and underline or box your answer.

1. __________

2. __________

3. __________

4. __________

5. __________

6. __________
Formulae:

\[ \tilde{V} = R \tilde{I} \]

\[ \tilde{V} = j\omega L \tilde{I} \]

\[ \tilde{V} = \frac{1}{j\omega C} \tilde{I} \]

\[ KCL \quad \sum_k \tilde{I}_k = 0 \]

\[ KV L \quad \sum_k \tilde{V}_k = 0 \]

\[ e^{j\omega t} = \cos \omega t + j \sin \omega t \]

\[ Z = \frac{\tilde{V}}{\tilde{I}} \]

\[ j = 0 + 1j = 1e^{j\frac{\pi}{2}} \]

\[ -j = 0 - 1j = 1e^{-j\frac{\pi}{2}} \]
1. (15%) For the three circuits shown, do the following:

(a) Write the appropriate differential equation and find the natural frequency or frequencies for each circuit.

(b) Write the general form of the solution for each of the circuits.
(You do not have to solve for any unknown constants).

Figure 1: Three circuits
2. (15%) Use the concept of phasors to add the sinusoids in each expression for \( v(t) \) and generate a single trigonometric expression.

(a) \( v(t) = 1 \cos(\omega t + \pi/4) + 2 \cos(\omega t + \pi/4) \)
(b) \( v(t) = 1 \cos(\omega t + \pi/4) + 3 \sin(\omega t + \pi/2 + \pi/4) \)

{Recall: \( \sin(\theta + \pi/2) = \cos(\theta) \)
3. (15%) The black box shown has some circuit element. We have the following measurements in steady-state:

\[ v_g(t) = 300 \cos(5000\pi t + \pi/2) \]
\[ i_g(t) = 6 \cos(5000\pi t + \pi) \]

(a) Determine the input impedance \( Z_{in} \).
(b) What type of circuit element might be in the black box?
4. (15%) In the circuit shown we have,

\[ i(t) = 1 \cos(t) \]

Given that the circuit is in steady-state, determine the voltage \( v(t) \).
Do the following:

(i) In circuits (a), $\omega = 1, C = 1$. Determine the input impedance $Z_{in}$.

(ii) In circuit (b) determine phasor voltage $\vec{V}_{out}$.

(iii) In circuit (c) determine phasor current $\vec{I}_2$. 

Figure 5: Three circuits

5. (15%)
6. (15%)

Determine the power associated with the 6 volt source.