9.8 Node Voltage Method.

![Circuit Diagram]

Node 1: 
\[-10.6 + \frac{V_1}{10} + \frac{V_1 - V_2}{1+j2} = 0\]

\[-10.6(1+j2) + 0.1V_1(1+j2) + V_1 - V_2 = 0\]

\[V_1(1+10.2) - V_2 = 10.6 + j21.2\] —— (1)

Node 2: 
\[\frac{V_2 - V_1}{1+j2} + \frac{V_2}{j5} + \frac{V_2 - 20I_x}{5} = 0\]

Controlling current \(I_x = \frac{V_1 - V_2}{1+j2}\)

Substitute for \(I_x\) in Node 2 equation:
\[\frac{V_2 - V_1}{1+j2} + j\frac{V_2}{5} + \frac{V_2}{5} - 20\frac{(V_1 - V_2)}{1+j2} = 0\]

\[5V_1 + (4.8 + j0.6)V_2 = 0\] —— (2)
Solving (1) and (2), get

\[ V_1 = 68.40 - j16.8 \text{ V} \]
\[ V_2 = 68 - j26 \text{ V} \]

\[ \therefore \text{ can be for all the } \text{ "branch currents" } \]
\[ I_a, I_x, I_c, I_b. \]

9.9 Mesh currents

Sections 4.5-4.7 mesh currents for resistive circuits.

\[ \text{Mesh 1: } 150 = (1+ j2)I_1 + (12 - j16) (I_1 - I_2) \]

\[ \text{Mesh 2: } 0 = (12 - j16)(I_2 - I_1) + (1 + j3)I_2 + 39I_x \]

Solve the 2 equations for \[ I_1, I_2 \]

Then can determine voltages across all the branch elements.
9.10 The Transformer

Applications: match impedances
Power systems: step down voltage

Simple application:

\[ V_S = (Z_S + R_1 + jwL_1) I_1 - jwM I_2 \]

Mesh 1

Mesh 2

\[ 0 = R_2 I_2 + I_2 Z_L - jwM I_1 + I_2 jwL_2 \]

To do:
(a) Find \( I_1, I_2 \)
(b) Find \( Z_{1b} \)
\[ V_s = \frac{Z_{11} I_1 - jWM I_2}{Z_{11} Z_{22} + W^2 M^2} \]

\[ I_1 = \frac{Z_{22}}{Z_{11} Z_{22} + W^2 M^2} V_s \]

\[ I_2 = \frac{jWM}{Z_{22}} I_1 \]

\[ Z_{11} = Z_s + jW L_1 \]

\[ Z_{22} = \text{self inductance} \]

\[ 0 = (R_2 + Z_L + jW L_2) I_2 - jWM I_1 \]

\[ V_s = Z_{int} = \frac{Z_{11} Z_{22} + W^2 M^2}{Z_{22}} = Z_{11} + \frac{W^2 M^2}{Z_{22}} \]

\[ \frac{V_s}{I_1} = Z_{int} = \frac{Z_s + Z_{ab}}{Z_s + Z_{2ab}} \]

\[ Z_{ab} = Z_{int} - Z_s = \frac{Z_{11} + W^2 M^2}{Z_{22}} - Z_s \]

\[ = R_1 + jW L_1 + \frac{W^2 M^2}{R_2 + Z_L + jW L_2} \]

\[ Z_{2ab} = \frac{Z_{2ab}}{Z_{22}} \]

Without the transformer, the load would be connected directly to the source, and the source would see \( Z_L \).

With the transformer we see a modified version of \( Z_L \).
Reflected Impedance

Third term \[ \frac{w^2 m^2}{R_2 + Z_L + jwL_2} \] called reflected impedance (Z_r)

It is the equivalent impedance of Z_L reflected to the primary side solely due to the mutual inductance.

\[ Z_r : \text{Consider } Z_L = R_L + jX_L \quad (\text{Note: } X_L \text{ is positive or negative}) \]

\[ Z_r = \frac{w^2 m^2}{\frac{R_2 + R_L + j(wL_2 + X_L)}{(R_2 + R_L)^2 + (wL_2 + X_L)^2}} \cdot \frac{(R_2 + R_L) - j(wL_2 + X_L)}{(R_2 + R_L) + j(wL_2 + X_L)} \]

\[ Z_{22} = \frac{w^2 m^2}{12a^2} \frac{(R_2 + R_L) - j(wL_2 + X_L)}{(R_2 + R_L)^2 + (wL_2 + X_L)^2} \]

Now \( Z_{22} \) is the "self-impedance" of the secondary circuit.

Eq. 9.68 shows that the self impedance of the secondary is reflected into the primary by a scaling factor:

\[ \frac{w^2 m^2}{12a^2} ; \text{ a sign of reactive component reversed.} \]
Thus, the transformer reflects the conjugate of the self-inductance of the secondary ($L_{2*}$) into the primary, times a scaling factor.
Ex: 9.13

\[ V_1, V_2 \text{ terminal voltages of transformer} \]

a) **Self Inductance of Primary**

\[ Z_{II} = 5m + j10n + 2m + j3600 = 7m + j3700 \]

b) **Self Inductance of Secondary**

\[ Z_{II} = 10m - j1600 + 8m - j2500 = 18m - j4100 \]

c) **Inductance reflected into the primary**

\[
Z_r = \left( \frac{1200}{1900 - j900} \right)^2 (900 + j1000)
\]

\[ = \left( \frac{8}{9} \right) (900 + j900) = 800 + j800 \]

So \( Z_{II} \) self inductance secondary reflected by \( \frac{8}{9} \) scaling factor.

d) **Inductance seen by \( a \) into primary \( \text{transformer} \)**

The transformer \( Z_{ab} = (200 + j3600) + 800 + j5111 \)

\[ = 1000 + j4401 \]
e) scaling factor by which $Z_{22}$ reflected = $8/9$

2) Thevenin equivalent wrt terminals C, D

$$V_{Th} = I_1 (j\omega M) = I_1 (j1200)$$

$$I_1 = \frac{300 \angle 0}{700 + j3700} = 79.67 \angle -79.29^\circ \ mA$$

$$\therefore V_{Th} = \left[ (79.67) \angle -79.29^\circ \times 10^{-3} \right] \cdot j1200$$

$$= 95.60 \angle 10.71^\circ \ V$$

$$Z_{Th} = \left(100 + j1600\right) + \frac{(\omega^2 M^2)}{|Z_{II}|^2} (100 - j3700)$$
Assessment problem 9.4.

\[ Z_L = 360 + j \omega (0.25) \]
\[ Z_3 = 180 + j0 \]
\[ (V_S)_{max} = 245.20 \quad \omega = 800 \text{ rad/s} \]

(a) **Reflected impedance**

\[ Z_r = \frac{W^2 M^2}{|Z_{22}|^2} \left[ (R_2 + R_0) - j (W L_2 + X_2) \right] \]

\[ M = k \sqrt{L_1 L_2} = 0.4 \sqrt{0.0625} = 0.1 \text{ H} \]
\[ \therefore W M = (800)(0.4) = 80 \]
\[ Z_{22} = 40 + j 800 (0.125) + 300 + j (800)(0.25) \]
\[ = (400 + j300) \Omega \]

\[ |Z_{22}| = \sqrt{400^2 + 300^2} = 500 \Omega \]
\[ Z_{22} = 400 - j300 \]
\[ \therefore Z_r = \frac{(80)^2}{(500)^2} \left[ (400 - j300) \right] = (10.24 - j76.8) \Omega \]
b) \[ I_1 \text{ - primary current} \]

\[
I_1 = \frac{245.20}{184 + 100 + j400 + Z_1} = 0.50 \angle -53.13^\circ
\]

\[ i_1(t) = 0.5 \text{ kA} (800t - 53.13^\circ) \]

c) \[ I_2 \text{ - secondary current} \]

\[
I_2 = \frac{jwM I_1}{Z_{22}} = 0.08 \angle 0^\circ A
\]

\[ i_2(t) = 80 \text{ kA peak mA} \]
9.11 Ideal Transformer

\[ M = k \sqrt{L_1 L_2} \]

1. Coefficient of coupling \( k = 1 \)

2. Self inductance \( L_1 = L_2 = \alpha \)

3. Coil loss due to parasitic resistance \( z_r \).

Recall:

\[
\begin{align*}
Z_{ab} &= R_1 + jwL_1 + \frac{w^2 M^2}{(R_2 + jwL_2 + Z_r) \cdot Z_r} \quad (9.64) \\
Z_{ab} &= Z_{11} + \frac{w^2 M^2}{Z_{22}} \cdot \frac{1}{Z_r} = R_1 + jwL_1 + \frac{w^2 M^2}{(R_2 + jwL_2 + Z_r) \cdot Z_r} \quad (9.64)
\end{align*}
\]

Now to see how \( Z_{ab} \) changes when \( k = 1, L_1, L_2 \rightarrow \alpha \):
Introduce:

\[ Z_{22} = R_2 + j (wL_2 + X_c) = R_{22} + jX_{22} \]

Then, rearrange (9.68)

\[ Z_{ab} = R_1 + \frac{w^2M^2 R_{22}}{R_{22}^2 + X_{22}^2} + j \left( wL_1 - \frac{w^2M^2 X_{22}}{R_{22}^2 + X_{22}^2} \right) \]

Like so:

\[ Z_{ab} = R_1 + jwL_1 + \frac{w^2M^2 R_{22}}{R_{22}^2 + X_{22}^2} \frac{R_{22} - jX_{22}}{R_{22} - jX_{22}} \]

\[ = R_1 + jwL_1 + \frac{w^2M^2 R_{22}}{R_{22}^2 + X_{22}^2} \frac{-jw^2M^2 X_{22}}{R_{22}^2 + X_{22}^2} \]

\[ Z_{ab} = R_1 + \frac{w^2M^2 R_{22}}{R_{22}^2 + X_{22}^2} + j \left( wL_1 - \frac{w^2M^2 X_{22}}{R_{22}^2 + X_{22}^2} \right) \quad (9.69) \]

\[ M = k \sqrt{L_{12}} \quad \text{eff. ampacity} \]

So, write \( X_{ab} = wL_1 \left( 1 - \frac{wL_2 X_{22}}{R_{22}^2 + X_{22}^2} \right) \)

\[ = wL_1 \left( \frac{R_{22}^2 + X_{22}^2 - wL_2 X_{22}}{R_{22}^2 + X_{22}^2} \right) \]

\[ = wL_1 \left( \frac{\overline{R_{22} + (wL_2 + X_c)(wL_1 + X_c - wL_1)}}{D} \right) \]
\[ X_{ab} = \frac{W_{L1}}{L} \left( \frac{R_{22}^2 + wL_z X_L + X_L^2}{R_{22}^2 + X_{22}} \right) \tag{9.71} \]

\[ = \frac{L_1}{L_2} \frac{X_L + (R_{22}^2 + X_L^2) / wL_z}{(wL_z/wL_2)^2 + [1 + (X_L/wL_2)^2]} \tag{9.72} \]

As \( L \to \infty \)

(9.72) reduces to

\[ X_{ab} = \left( \frac{L_1}{L_2} \right)^2 X_L \]

Similarly, \( R_{ab} = \left( \frac{L_1}{L_2} \right)^2 R_{22} \)

\[ R_{ab} = R_1 + \left( \frac{L_1}{L_2} \right)^2 R_{22} + \left( \frac{L_1}{L_2} \right)^2 (R_c + jX_L) \tag{9.75} \]

Check with (8.1) matches that

\[ X_{ab} \text{ where } L_1^2, L_1, L_2 \to \infty, \]

transformer reacts secondary winding resistance & load influence to the primary by a scaling factor \( \left( \frac{L_1}{L_2} \right)^2 \)
Determining voltage or current ratios.

\[ V_2 = j\omega M I_1 \]

\[ J_1 = \frac{V_1}{j\omega L_1} \]

\[ V_2 = \frac{M}{L_1} V_1 \]

\[ = \sqrt{\frac{L_2}{L_1}} V_1 \]

\[ = \sqrt{\frac{N_2^2 R}{N_1^2 R}} V_1 \]

\[ = \frac{N_2}{N_1} V_1 \]

\[ \therefore \frac{V_2}{V_1} = \frac{N_2}{N_1} \]
Symbol for ideal transformer

\[ V_2 = \frac{N_2}{N_1} V_1 \quad N_1 I_1 = -N_2 I_2 \]

Rules:
- If coil vitiates both in- or react, use the plus. Otherwise negative.
- If coil currents \( I_1, I_2 \) both clockwise, or out was - sign, otherwise + ve sign.
Ex. 9.14

Find steady state (a) \( i_1(t) \)  (b) \( V_1(t) \)  (c) \( i_2(t) \)  (d) \( V_2(t) \)

\[
2500 L_0 = (0.25 + j 2) I_1 + V_1 \\
V_1 = 10 V_2 = 10 \left( 0.2375 + j 0.05 \right) I_2 \\
I_2 = 10 I_1 \\
\therefore V_1 = 100 \left( 0.2375 + j 0.05 \right) I_1 \quad \Rightarrow \quad I_1 = 100 L_0 \cdot 16.26^\circ A \\
\therefore i_1(t) = 100 \cos (400t - 16.26^\circ) A
\]
6) \[ V_1 = 2500\angle 0^\circ \cdot (0.25 + j\ 2) \cdot (100\angle -16.26^\circ) \]
\[ = 2500\angle 0^\circ - 80 - j\ 185 \]
\[ = 2420 - j\ 185 = 2427.06 \angle -4.37^\circ \]
\[ \therefore v_1(t) = 2427.06\ \cos(400t - 4.37^\circ)\ \text{V} \]

c) \[ I_2 = 10 I_1 = 1000\angle -16.26^\circ\ \text{A} \]
\[ \therefore i_2(t) = 1000\ \cos(400t - 16.26^\circ)\ \text{A} \]

\[ V_2 = 0.1 V_1 = 242.71 \angle -4.37^\circ\ \text{V} \]
\[ \therefore v_2(t) = 242.71\ \cos(400t - 4.37^\circ)\ \text{V} \]