Chapter 11  Balanced Three-Phase Circuits

Generation, transmitting & distributing of large amounts of power accomplished with three-phase circuits.

We consider sinusoidal steady-state behavior of balanced three-phase circuits. (study of unbalanced 3-phase rel'n on balanced 3-phase).

![Diagram of three-phase circuit with labeled parts]

We omit primary transformers & transmission lines. This simplifies the discussion & does not affect the understanding.

So we want to handle large blocks of electric power.

We show the type of circuit used to deliver power to an entire residential subdivision.
11.1 Balanced 3-Phase

A set of balanced 3-phase voltages consists of 3 sinusoidal voltages—identical amplitudes and frequencies—but out of phase by 120°.

![Diagram showing phase relationships]

Standard practice: Use a-phase as reference.

Only 2 possible phase relationships.

\[
\begin{align*}
V_a &= V_m \angle 0^\circ \\
V_b &= V_m \angle -120^\circ \\
V_c &= V_m \angle +120^\circ
\end{align*}
\]

\[\text{or } V_c = V_m \angle +240^\circ \text{ or } V_m \angle -120^\circ\]

Note: When circuits operated in parallel, must have same phase sequence.

abc or positive phase sequence

acb or negative phase sequence
Another fundamental fact: for a set of balanced 3-phase:

For (11.1) or (11.2)

\[ V_a + V_b + V_c = 0 \]

and it follows that \[ v_a(t) + v_b(t) + v_c(t) = 0 \]

Another fundamental fact:

If we know the phase sequence and 1 voltage in the set, we know the entire set. \( \therefore \) For the balanced 3-phase, we focus on determining voltage (or current) in 1 phase. (\( \therefore \) then we know the others).

11.2 3-Phase Voltage Sources

Fig. 11.3
2 ways of interconnecting the separate phase windings to form a 3-phase source: Y or Δ configuration.

And where winding impedances need to be factored, each branch is shown as: $V_a - Ra JWL_a$
3-phase source & load connections:

Source
\[ Y \]
\[ \Delta \]
\[ Y \]
\[ \Delta \]

Load
\[ Y \]
\[ \Delta \]
\[ Y \]
\[ \Delta \]

Figure 11.1

We begin by analyzing the Y-Y connection. Others can be reduced to Y-Y, so analysis of Y-Y is key.

11.3 Analysis of Y-Y circuit

\[ V_{a} \rightarrow \frac{Z_{k}}{a} \rightarrow V_{N} \rightarrow \frac{I_{a}}{a} \rightarrow A \]

\[ V_{b} \rightarrow \frac{Z_{k}}{b} \rightarrow V_{N} \rightarrow \frac{I_{b}}{b} \rightarrow B \]

\[ V_{c} \rightarrow \frac{Z_{k}}{c} \rightarrow V_{N} \rightarrow \frac{I_{c}}{c} \rightarrow C \]

\[ Z_{0}, \text{ internal impedances} \]

\[ Z_{L}, \text{ line impedances} \]

\[ Z_{d}, \text{ load impedances} \]
voltage in the set, we know the entire set. Thus for a balanced three-phase system, we can focus on determining the voltage (or current) in one use once we know one phase quantity, we know the others.

E: Assess your understanding of three-phase voltages by trying to solve Problems 11.2 and 11.3.

## 2 Three-Phase Voltage Sources

A three-phase voltage source is a generator with three separate windings distributed around the periphery of the stator. Each winding corresponds to one phase of the generator. The rotor of the generator is an electromagnet driven at synchronous speed by a prime mover, such as a gas turbine. Rotation of the electromagnet induces a sinusoidal voltage in each winding. The phase windings are designed so that the induced voltages in them are equal in amplitude and out of phase with each other by 120°. The phase windings are stationary with respect to the rotating electromagnet, so the frequency of the voltage produced in each winding is the same. Figure 11.3 shows a sketch of a two-winding three-phase source.

There are two ways of interconnecting the separate phase windings to form a three-phase source: in either a wye (Y) or a delta (Δ) configuration. Figure 11.4 shows both, with ideal voltage sources used to model the windings of the three-phase generator. The common terminal in the wye-connected source, labeled n in Fig. 11.4(a), is called the neutral terminal or neutral. The neutral terminal may or may not be available for external loads, but in most cases, the impedance of each phase winding is so small compared to other impedances in the circuit that we need not account for it when analyzing the generator; the model consists solely of ideal voltage sources as in Fig. 11.4. However, if the impedance of each phase winding is nonnegligible, we place the winding impedance in series with an ideal voltage source. All windings on the machine are of the same type, so we assume the winding impedances to be identical. The Y-Δ model of such a machine, in which the winding resistance, and the inductive reactance of the winding, is shown. Three-phase sources and loads can be either Y-connected or Δ-connected, the basic circuit in Fig. 11.1 represents four different arrangements:

<table>
<thead>
<tr>
<th>Source</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Y</td>
<td>Δ</td>
</tr>
<tr>
<td>Δ</td>
<td>Y</td>
</tr>
<tr>
<td>Δ</td>
<td>Δ</td>
</tr>
</tbody>
</table>

by analyzing the Y-Y circuit. The remaining three arrangements can be deduced to a Y-Y equivalent circuit, so analysis of the Y-Y circuit is sufficient for solving all balanced three-phase arrangements. We then illustrate the Δ arrangement and leave the analysis of the Y-Δ arrangements to you in the Problems.

Figure 11.3 A sketch of a three-phase voltage source.

Figure 11.4 The two basic connections of an ideal three-phase source. (a) A Y-connected source. (b) A Δ-connected source.
Single node relay equation sufficient to describe arcing:

Using (1) as the reference node, we get

\[
\frac{V_n}{Z_0} + \frac{V_n-V_{an}}{Z_a+Z_ea+Z_ga} + \frac{V_n-V_{bn}}{Z_b+Z_eb+Z_gb} + \frac{V_n-V_{cn}}{Z_c+Z_ec+Z_gc} = 0
\]  (11.5)

This is the general equation for any Y-Y configuration. Can simplify (11.5) significantly with the following:

(a) \( V_{an}, V_{bn}, V_{cn} \) from a set of balanced 3-phase voltages.
(b) Source voltage impedances same \( Z_ga = Z_gb = Z_gc \).
(c) Line impedances \( Z_{ea} = Z_{eb} = Z_{ec} \) same.
(d) Impedance of each phase of the load same \( Z_e = Z_e = Z_e \).

\[ V_n \left( \frac{1}{Z_0} + \frac{3}{Z_\phi} \right) = \frac{V_{an} + V_{bn} + V_{cn}}{Z_\phi} \]  (11.6)

where \( Z_\phi = Z_a + Z_ea + Z_ga = Z_b + Z_eb + Z_gb = Z_c + Z_ec + Z_gc \).
A.H.S of (11.6) = 0 since balanced 3-phase.

\[ V_n = 0 \]  

(\text{voltage between any two - no difference in potential}).

\[ \therefore \text{current in neutral conductor} = 0 \text{A} \]

Hence we may either remove the neutral conductor or simply replace it with a s.c.

For the equivalent used when modeling balanced 3-phase circuits.

Now let's look at the 3 line currents:

\[ I_{A} = \frac{V_{an}}{Z_{\phi}} \]  

\[ I_{B} = \frac{V_{bn}}{Z_{\phi}} \]  

\[ I_{C} = \frac{V_{cn}}{Z_{\phi}} \]  

\[ Z_{\phi} = Z_A + Z_{ea} + Z_{ga} \]

Note: Currents form a balanced set of 3-phase currents.

In each line: equal in amplitude, frequency = 120° out of phase with the other 2 line currents.
This current

Thus, if we know $I_{aA}$ in the phase sequence, we can easily find $I_{bb}$, $I_{cc}$.

Equation (11.6) describes:

\[ V_{an} \]

Fig. 11.7

Single phase equivalent circuit in a balanced 3-phase.

Once we solve this, we can easily write down voltages and currents in the other two phases.

\underline{Word of caution:} Current in the neutral conductor is $I_{aA}$, not the same as current in the neutral conductor of the balanced 3-phase:

\[ I_0 = I_{aA} + I_{bb} + I_{cc} \]

which sum to zero.
Once we know the line current, we can calculate any voltage.

We are interested in line-to-line voltages $V_{AB}, V_{BC}, V_{CA}$ and line-to-neutral voltages $V_{AN}, V_{BN}, V_{CN}$

(We establish relationships at the load terminals, but they are similar at source terminals)

\[
V_{AB} = V_{AN} - V_{BN} \\
V_{BC} = V_{BN} - V_{CN} \\
V_{CA} = V_{CN} - V_{AN}
\]

Assume positive or $ABC$ sequence:

\[
V_{AN} = V_\phi L^0 \\
V_{BN} = V_\phi L -120^\circ \\
V_{CN} = V_\phi L +120^\circ
eq V_\phi - magnitude of line-to-neutral voltage.
\[ V_{AB} = V_\phi \angle 0^\circ - V_\phi \angle -120^\circ \]
\[ V_{BC} = \sqrt{3} V_\phi \angle -30^\circ \]
\[ V_{CA} = \sqrt{3} V_\phi \angle -150^\circ \]

So:
1. Mag. line-to-line = \( \sqrt{3} \) line-to-neutral
2. Line-to-line form a balanced 3-phase
3. Line-to-Line lead line-to-neutral voltage by 30°

To clarify:

**Line voltage refers to voltage across any pair of line phases**, not a single phase.

**Line current** - current in a single line phase.

Typically, all voltages and currents given in RMS.

When voltage ratings are given - refers to the line voltage.

Ex: 345 kV \( \Rightarrow \) line-to-line voltage is 345,000 with RMS.
Example 11.1  Y-Y circuit analysis.

Balanced 3-phase Y-connected generator connected to Y-connected load.

Single-phase equivalent circuit.

a) Single-phase equivalent circuit.
b) Line currents $I_{AA}$...
c) 3 phase voltages at the loads. $V_{an}$, ...
d) Line voltages $V_{AB}$, ... at load terminals.
e) Phase voltages $V_{an}$, ... at generator terminals.
f) Line voltage $V_{ab}$, ... at "...
g) Neutral for negative phase.
b) \[ I_{AA} = \frac{120/0}{(0.2+0.8+39)+j(0.5+1.5+28)} = 2.4/36.87 \]

For positive phase sequence:

\[ I_{BB} = 2.4 \begin{pmatrix} -151.87 \end{pmatrix} A \]
\[ I_{CC} = 2.4 \begin{pmatrix} 18.13 \end{pmatrix} A \]

c) \[ V_{AN} = (39+j28) \cdot (2.4/136.87) = 115.22/121.19 V \]
\[ V_{BN} = 115.22/121.19 V \]
\[ V_{CN} = 115.22/118.51 V \]

d) \[ V_{AB} = (\sqrt{3}/130\) \cdot V_{AN} = 199.58/28.81 \]
\[ V_{BC} = (\sqrt{3}/30\) \quad 199.58/91.19 V \]
\[ V_{CA} = 199.58/148.81 V \]

e) Phase angle at source terminal

\[ V_{AN} = 120 - (10+0.5)(2.4/136.87) \]
\[ = 120 - 1.29 \begin{pmatrix} 51.33 \end{pmatrix} = 118-0.67 \]
\[ V_{BN} = 118.90/120.32 V \]

For positive phase sequence:

\[ V_{BN} = 118.90/119.08 \]
f) Line voltage \( V_{ab} \) at generator terminals:

\[
V_{ab} = (\sqrt{3} \angle 30^\circ) \ V_{an} = (\sqrt{3} \angle 30^\circ) \ (116.9 \angle -0.32^\circ)
= 205.94 \angle 29.68^\circ
\]

\[
V_{bc} = 205.94 \angle -90.32^\circ V
\]

\[
V_{ca} = 205.94 \angle 149.68^\circ V
\]

g) Repeat for negative phase

Assessment problem (1.1)

(11.1) Balanced 3-phase.

\[
\tilde{V}_{an} = 240 \angle -30^\circ V
\]

The phase sequence.

\[
\tilde{V}_{bc} = ?
\]

\[
V_{ab} = \tilde{V}_{an} - \tilde{V}_{bn} = \sqrt{3} V_\phi L 30^\circ
\]

If \( \tilde{V}_{an} = V_\phi L 0 \), then \( V_{ab} = \sqrt{3} V_\phi L 30^\circ \).

Given \( \tilde{V}_{an} = V_\phi L 240^\circ \), \( \Rightarrow V_{ab} = \sqrt{3} V_\phi L 240^\circ \).

\[
\therefore V_{bc} = \sqrt{3} 240 L -120^\circ V = 415.69 L -120^\circ V
\]
A balanced 3-phase negative phase

\[ V_{CN} = 450 \angle -25^\circ \]

\[ V_{AB} = ? \]

We know thus:

If \( V_{CN} = V_{\phi} \), then

\[ V_{CN} = V_{\phi} \angle -120^\circ \]

\[ V_B = V_{\phi} \angle 120^\circ \]

Also:

\[ V_{AB} = \sqrt{3} V_{\phi} \angle -30^\circ \] (negative phase)

But \( V_{BN} = V_{CN} = 450 \angle -25^\circ \)

\[ V_{AN} = 450 \angle 95^\circ \]

\[ V_{AB} = \sqrt{3} \times 450 \angle 65^\circ \]