Chapter 10: Sinusoidal Power Calculations

10.1 Instantaneous Power
10.2 Airflow and Reactor ...
10.3 Power in Reverse ...
10.4 Complex Power ...
10.5 Power Calculations ...
10.6 Power Triangle ...

Chapter 9: Electric circuits driven by sinusoidal sources
Chapter 10: Instantaneous Power in such circuits

Why? Sinusoidal sources are the predominant means of providing electric power in the world.

We have solar power, wind power, nuclear ...

Understanding fundamentals of distributing electricity:

Generation, Transmission, Distribution
Convention for $v, i$

$$p(t) = v(t) \cdot i(t)$$

If $p(t) > 0$ circuit absorbing power

If $p(t) < 0$ circuit delivering power.

Example

$$\begin{align*}
1.5 \text{ volts} &= E \text{ volts} \\
1.5 \text{ mA} &= i
\end{align*}$$

$$R = 1 \text{ KΩ}$$

$$v = 1.5$$

$$i = \frac{1.5}{1000} = 1.5 \text{ mA}$$

$$\therefore \text{ for } E, \quad p = 1.5 \times 1.5 \times 10^{-3} = 2.25 \text{ mW}$$

$$\text{for } R, \quad p = 1.5 \times 1.5 \times 10^{-3} = 2.25 \text{ mW}$$
Diagram of an electric power system, generation system in black
In circuits, in sinusoidal steady-state we have

\[
\begin{align*}
    &i(t) = I_m \cos(\omega t + \theta_i) \\
    &v(t) = V_m \cos(\omega t + \theta_v)
\end{align*}
\]  

(10.2)  

(10.3)  

\(\theta_i\) - current phase angle  
\(\theta_v\) - voltage phase angle  

Want to choose any convenient reference for zero time.  

Convenient to choose current passing through maximum.  

So we shift both \(i(t)\) and \(v(t)\) by \(\theta_i\), get

\[
\begin{align*}
    i(t) &= I_m \cos(\omega t + \theta_v - \theta_i) \\
    v(t) &= V_m \cos(\omega t + \theta_v - \theta_i)
\end{align*}
\]  

(10.4)  

(10.5)  

With the sign convention shown, instantaneous power

\[
p(t) = v(t)i(t) \quad \text{(watts)}
\]

\[
= V_m I_m \cos(\theta_v - \theta_i) 
\]

Using \(\cos x \cos y = \frac{1}{2} \cos(x-y) + \frac{1}{2} \cos(x+y)\), get

\[
p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v - \theta_i)
\]
Now using:

\[ \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \]

get,

\[ P(t) = \frac{V_m I_m}{2} \cos (\omega t + \theta_v - \theta_i) + \frac{V_m I_m}{2} \cos \omega t - \frac{V_m I_m}{2} \sin (\omega t + \theta_v - \theta_i) \sin \omega t \]

(10.8)

\[ \theta_v = 60^\circ \]
\[ \theta_i = 0 \]

---

Note:

(a) Frequency of instantaneous power is same as that of voltage or current.

(b) Instantaneous power can be negative for a portion of each cycle even if the average is positive.

Negative power means that energy (stored in L, C) is being extracted.
0.2 .

**Active + Reactive Power**

\[
p(t) = \frac{V_m J_m \cos(\theta_t - \theta_i)}{2} + \frac{V_m J_m \cos(\theta_t - \theta_i) \cos \omega t}{2}
\]

\[
- \frac{V_m J_m \sin(\theta_t - \theta_i) \sin \omega t}{2}
\]

\[
p(t) = P + P \cos \omega t - Q \sin \omega t
\]

\[
p = \int_{-T/2}^{T/2} p(t) \, dt = \frac{V_m J_m}{2} \cos(\theta_t - \theta_i)
\]

**Called Average Power (over a period)**

**Reactive Power**

**Active Power**

\[
P = \frac{1}{T} \int_{-T/2}^{T/2} p(t) \, dt = \frac{V_m J_m}{2} \cos(\theta_t - \theta_i)
\]

**Since:**

\[
p(t) = A + A \cos 2\omega t - B \sin 2\omega t
\]

\[
\int_{-T/2}^{T/2} p(t) \, dt = A + \frac{2}{T} \int_{-T/2}^{T/2} A \cos 2\omega t \, dt
\]

\[
= A + \frac{2}{2T} \left[ \frac{\sin 2\pi \frac{1}{2} - 0}{2\pi} \right] + \frac{2}{2T} \left[ \cos 2\pi \frac{1}{2} - 1 \right]
\]

\[
= A
\]

\[
W = \frac{2\pi}{T}
\]

\[
2\omega = \frac{2\pi}{T/2}
\]
can better understand (11.9) through the given example.

\[ R : \quad \theta_v = \theta_i \]

\[ p(t) = \frac{V_m J_m}{2} \left( \frac{\cos(\theta - \theta_i)}{2} \right) \sin^2 \omega t + \frac{V_m J_m}{2} \left( \frac{\sin(\theta - \theta_i)}{2} \right) \cos^2 \omega t - \frac{V_m J_m}{2} \sin(\theta - \theta_i) \sin \omega t \]

\[ \therefore p(t) = P + P \cos 2\omega t \]

**Note:** Instantaneous power never negative for \( R \). \( p(t) \) always \( \geq 0 \).

\[ \therefore \text{power can be extracted from an AC network}. \]

\[ L : \quad I = \frac{V}{j\omega L} \Rightarrow i(t) \text{ lags } v(t) \text{ by } 90^\circ \]

\[ \therefore \theta_i = \theta_v - 90^\circ \]

\[ \therefore p(t) = 0 + 0 - Q \sin 2\omega t \]

\[ Q = \frac{V_m J_m}{2} \sin(\theta - \theta_i) \]
In an inductive circuit,
\[ P = 0, \quad (\text{Re} = 0) \]

No transformation of energy from electric to non-electric.

Instantaneous power at the terminals continually exchanged between the circuit & source at a frequency \( 2\omega \).

In other words: When \( p \) positive, energy is being stored in the magnetic field.

When \( p \) negative, energy being extracted from the magnetic field.

A measure of power associated with inductive elements called reactive power \( Q \).

**Note:** Both \( P, Q \) have same dimension.

To distinguish, we use \( \text{watts} \) for \( P \),

\( \text{vars} \) (volt ampere reactive) for \( Q \).
\[ I = \frac{V}{Z} = j\omega CV \]

\[ i(t) \quad \text{leads} \quad v(t) \quad \text{by} \quad 90^\circ \]

\[ \theta_i = \theta_v + 90^\circ \]

\[ p(t) = -Q \sin 2\pi nt \]

Again: \[ P_r e = 0 \]

\[ Q = \frac{V_m \sin \left( \frac{\theta_v - \theta_i}{2} \right)}{2} \]

Energy is periodically exchanged between electric and magnetic forms.

\[ Q \quad \text{assumed} = 1 \]
Power Factor

Angle $(\theta_r - \theta_i)$ occurs in both average & reactive power

$$P_{th} = P + P_{loss} \cos \omega t - Q \sin \omega t$$

$$P = \frac{V_m I_m \cos (\theta_r - \theta_i)}{2}$$

$$Q = \frac{V_m I_m \sin (\theta_r - \theta_i)}{2}$$

$$P_f = \cos (\theta_r - \theta_i)$$

$$Q_f = \sin (\theta_r - \theta_i)$$

We speak of lagging power factor (current lags voltage) \((L.P.F.)\) \((L.P.F.)\)

& leading power factor (current leads voltage) \((L.P.F.)\) \((L.P.F.)\)

**Example 10.1**

\[ v = 100 \cos (\omega t + 15^\circ) \text{ V} \]

\[ i = 4 \sin (\omega t - 15^\circ) \text{ A} \]

\[ P = \frac{1}{2} (100)(4) \cos [(15 - (-105))] = -100 \text{ W (since circuit box delivering power)} \]

\[ Q = \frac{1}{2} (100)(4) \sin [(15 - (-105))] = 173.2 \text{ VAR (since Q > 0 circuit absorbs VARs)} \]
AP 10.1 (HW 6)

a) \( v = 100 \, \text{m/s} \left( \text{wt} - 45^\circ \right) \)
\( \omega = 20 \, \text{rad/s} \left( \text{wt} + 15^\circ \right) \)

\( P = 500 \, \text{W} \) \( (\because \text{pwr from from } \text{A} \rightarrow \text{B}) \)
\( Q = -866.03 \, \text{VARs} \) \( (\because \text{vans from } \text{B} \rightarrow \text{A}) \)

b) \( v = 100 \, \text{m/s} \left( \text{wt} - 45^\circ \right) \) \( v \)
\( \omega = 20 \, \text{rad/s} \left( \text{wt} + 15^\circ \right) \) \( \omega \)

\( P = 866.03 \, \text{W} \)
\( Q = 500 \, \text{VARs} \)

<table>
<thead>
<tr>
<th>( A \rightarrow B )</th>
<th>( P )</th>
<th>( Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A \rightarrow B )</td>
<td>500 W</td>
<td>866.03 VARs</td>
</tr>
<tr>
<td>( B \rightarrow A )</td>
<td>-500 W</td>
<td>-866.03 VARs</td>
</tr>
</tbody>
</table>
Appliance Ratings

Power quantifies power needs of household appliances

Annual KW-Hrs consumption given Table 10.1

Example: coffee maker estimated annual consumption 140 kWh

\[ \text{Pare} = 1.2 \text{ kW} \]

\[ \therefore \text{assumed to operate} \frac{140}{1.2} = 116.67 \text{ hrs/year} \]

\[ \text{or about 19 minutes/day}. \]

Ex: 10.2

Following 4 appliances (120 volts) operating at same time:

- Coffee maker: 1200 W
- Egg cooker: 516 W
- Fry pan: 1196 W
- Toaster: 1146 W

(Pare) total: 4058 W

\[ \therefore \text{Jeff} = \frac{4058}{120} \approx 33.82 \text{A} \]
10.5 \[ \text{rms value} = \frac{\text{Power}}{\text{rms value}} \]

Recall: \[ \text{rms value} = \sqrt{\frac{1}{T} \int_0^T V_m \cos^2(\omega t) \, dt} \]

\[ V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T V_m \cos^2(\omega t) \, dt} = \frac{V_m}{\sqrt{2}} + \underbrace{\text{for}}_{R} \]

Consider:

\[ v(t) = V_m \cos(\omega t + \theta V) \]

Find average power delivered to resistor:

\[ P = \frac{1}{T} \int_0^T p(t) \, dt = \frac{1}{T} \int_0^T \frac{V_m^2 \cos^2(\omega t + \theta V)}{R} \, dt \]

\[ = \frac{1}{TR} \frac{V_m^2}{2} \int_0^T (1 + \cos 2(\omega t + \theta V)) \, dt \]

\[ = \frac{V_m^2}{RT^2} \left[ t + \frac{1}{2w} \sin(2(\omega t + \theta V)) \right]_0^T \]

\[ = \frac{V_m^2}{RT^2} \left[ T + \frac{1}{2w} \sin \left(\frac{2\pi T}{T} \right) - \frac{1}{2w} 0 \right] \]

\[ P_{\text{ave}} = \frac{V_m^2}{RT^2} \left[ T \right] = \frac{V_m^2}{2R} = \frac{V_{\text{rms}}^2}{R} \]
Equivalently:

\[
\text{Peak} = \sqrt{2} \text{ RMS}
\]

**RMS value also referred to as **effective value**

RMS value has an interesting property.

Consider the following 2 circuits:

- **(a)** \( V_s = 100 \text{ V (rms)} \)
- **(b)** \( V_s = 100 \text{ V (dc)} \)

\( R \) the same in both circuits.

and voltage connected for same time period \( T \).

The RMS value of an unidirectional source delivers the same energy to \( R \) as does a DC source of the same value.

That is, 100V RMS delivers same energy to \( R \) as a DC voltage source of the same value 100V.
Recall

\[ P_{ac} = \frac{V_m J_m \cos(\theta_v - \theta_i)}{2} \]

\[ = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i) \]

\[ = V_{rms} I_{rms} \cos(\theta_v - \theta_i) \]

\[ \text{for } R: \quad \theta_v = \theta_i; \]

\[ \therefore P_{ac} = V_{eff} I_{eff} \cos(\theta_v - \theta_i) \]

\[ \text{Similarly: } Q = V_{eff} I_{eff} \sin(\theta_v - \theta_i) \]

The effective value of sinusoidal voltage sources is commonly used in power calculations; it is so widely used that voltage and current ratings of equipment are often given in terms of rms values.

Ex: Rating of residential electrical wiring often 120V/240V rms.

Apparatus such as lamps, toasters, ovens carry rms rating.

Such as: 120V, 180W light bulb \( \Rightarrow R = \frac{120^2}{180} = 80.0 \) \( \Omega \)

\[ V_{rms} = R I_{rms} \]

\[ V_{rms} \text{ draws } I_{rms} = \frac{100}{120} = \frac{P}{V_{rms}} \]

Also \( I_{rms} = \frac{120}{144} \)
Example 10.3

\( v(t) \) is sinusoidal, max. amp. = \( V_m = 625 \text{V} \)

\( \text{a) Power delivered to resistor} = \frac{V_m^2}{R} = \left( \frac{625}{\sqrt{2}} \right)^2 \frac{1}{50} = 3906.25 \text{W} \)

\( \text{b) rms current} \quad I_{\text{rms}} = \frac{V_m \phi}{R} = \frac{625}{50 \sqrt{2}} = 12.5 \text{A} \quad I_{\text{rms}} = \frac{12.5}{\sqrt{2}} \)

\[ \therefore \text{Pare} = I_{\text{rms}}^2 R = \left( \frac{8.84}{\sqrt{2}} \right)^2 \times 50 \]

\[ = 3906.25 \]

AP 10.3

\( I_P = 180 \text{mA} \)

\( \text{and Power delivered to a single resistor} \)

\( P_{\text{eff}} = v(t) i(t) = i^2(t) R \)

\[ \text{Pare} = \frac{1}{T} \int_0^T i^2(t) R \, dt \]

\[ = R \frac{1}{T} \int_0^T i^2(t) \, dt \]

\( I_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T i^2(t) \, dt} \)
In this equation,
- the first term is constant,
- the 2\textsuperscript{nd} and 3\textsuperscript{rd} show that instantaneous power has twice frequency of the voltage or current.
- It has negative for some portion of cycle. That means, energy stored in the inductors or capacitors is now being extracted.

![Graph showing power versus time](image)

**Average and Reactive Power.**

We can divide the instantaneous power (last equation) in three terms.

\[ p = P + P \cos 2\omega t - Q \sin 2\omega t \]  
(1.4)

where \( P \) is called the **average power or real power**

\[ P = \frac{V_m I_m}{2} \cos(\theta_i - \theta_e) \text{ [W]} \]  
(1.5)

\( Q \) is called the **reactive power**

\[ Q = \frac{V_m I_m}{2} \sin(\theta_i - \theta_e) \text{ [VAR]} \]  
(1.6)

The average power be represented in the following form too

\[ P = \frac{1}{T} \int_{t_0}^{t_0 + T} pdt \]

\( T \) is period of the sinusoidal function.
Example: \( v = 100 \cos(\omega t + 15^\circ) \) V.
\[ i = 4 \sin(\omega t - 15^\circ) \] A

Let's convert current \( i(t) \) in cos function

\[ i = 4 \cos(\omega t - 15^\circ - 90^\circ) = 4 \cos(\omega t - 105^\circ) \]

the average power or real power will be

\[ P = \frac{V_m I_m \cos(\theta_v - \theta_i)}{2} \]
\[ P = \frac{(100)(4)}{2} \cos(15 - (-105)) = -100 \text{ W} \]

(Network inside the box is delivering average power to the terminal)

The reactive power

\[ Q = \frac{V_m I_m \sin(\theta_v - \theta_i)}{2} \]
\[ Q = \frac{(100)(4)}{2} \sin(15 - (-105)) = 173.21 \text{ VAR} \]

(Inside the box is absorbing magnetizing vars at its terminal.)

The instantaneous power will be

\[ p = -100 - 100 \cos 2\omega t - 173.21 \sin 2\omega t \]
10- Sinusoidal Steady-State Power Calculations

- All electrical energy is supplied in the form of sinusoidal voltages and currents.
- Our interest is the average power delivered to or supplied from a pair of terminals as a result of sinusoidal voltages and currents

Instantaneous Power

\[ i \\
+ \\
v \\
- \\
\downarrow \]

Box

Instantaneous Power is the power at any instant of time is

\[ p = vi \]

where

\[ v = V_m \cos(\omega t + \theta_v) \]
\[ i = I_m \cos(\omega t + \theta_i) \]

Let's shift both voltage and current by \( \theta_i \),

\[ v = V_m \cos(\omega t + \theta_v - \theta_i) \]
\[ i = I_m \cos(\omega t) \]

Therefore, instantaneous power becomes

\[ p = vi = V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos\omega t \] (1.1)

To simplify this, we can use the following trigonometric identity

\[ \cos \alpha \cos \beta = \frac{1}{2} \cos(\alpha - \beta) + \frac{1}{2} \cos(\alpha + \beta) \]

\[ p = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(2\omega t + \theta_v - \theta_i) \] (1.2)

We know that

\[ \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \]

Therefore, instantaneous power

\[ p = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \cos 2\omega t - \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \sin 2\omega t \] (1.3)
RMS value determined Ex: 9.4.

\[
\int_0^T = 4 \int_0^{T/4} \frac{16 I_p^2}{T^2} t^2 dt = \frac{I_p^2 T}{3}
\]

\[
\frac{1}{T} \int_0^T = \frac{I_p^2 T}{T^3} = \frac{I_p^2}{3}
\]

\[
I_{\text{rms}} = \sqrt{\frac{I_p^2}{3}} = \frac{I_p}{\sqrt{3}} = \frac{0.18}{\sqrt{3}}
\]

\[P_{\text{real}} = I_{\text{rms}}^2 R = \left( \frac{0.0324}{3} \right) (5000) = 5.4 \text{ W}
\]

10.4 Complex Power:

Need to define complex power:

\[S = P + jQ \quad (10.23)\]

From \(S\) we can get \(P\) or \(Q\).

Power quantities in units: (all share same dimension).

\[\text{Complex power} \quad \text{watt-amps}
\]

\[\text{Average power} \quad \text{watts}
\]

\[\text{Reactive power} \quad \text{var}
\]
Geometric interpretation

\[ P, Q, \, |S| \text{ sides of a right triangle.} \]

\[ S = |S| e^{i\theta} \]

\[ |S| = \text{effective power} \]

\[ P = \text{average power} \]

\[ Q = \text{reactive power} \]

\[ \theta \]

\[ \text{Fig. 10.9 Power triangle} \]

**Note:**

\[ \tan \theta = \frac{Q}{P} \]

\[ = \frac{(V_m I_m/2) \sin (\theta_v - \theta_i)}{(V_m I_m/2) \sin (\theta_v - \theta_i)} \]

\[ = \tan (\theta_v - \theta_i) \]

\[ \therefore \theta = (\theta_v - \theta_i) \text{ which is the power factor angle.} \]

So: given \(|S|, P, Q, \phi\) triangle

From any 2 can determine the other two.

\[ |S| = \sqrt{P^2 + Q^2} \text{ called apparent power} \]

**Note:** The apparent power, or volt-amperes requirement of a device (designed to convert electric energy to non-electric form) is more important than the average power requirement.
Part represents the useful output of the energy-converting device. However, the apparent represents the volt-amp capacity required to supply the average power.

\[ \theta \]

Unless \( \theta = 0 \) (\( p = 1 \)) \( \Rightarrow \theta = 0 \) a device purely resistive, the volt-amp capacity read by the device is larger than the average power used. (As we will see in Ex: 10.6 makes sense to operate devices at a PF closer to 1.)

Practical

Many home devices - refrigerators, fans, AC, fluorescent lamps, washing machines - also most industrial loads operate at a lagging PF.
Example 10.44

![Diagram showing electrical load and power triangle]

Electrical load operates at 240 V rms

" " absorbs average power 8 kW

" " lagging* \[ PF = 0.8 \] \[ \cos \theta = 0.8 \]

\[ \sin \theta = 0.6 \]

a) Complex power = ?

**PF lagging ⇒ load inductive**

From power \( \Delta \),

\[ P = |S| \cos \theta \]

\[ Q = |S| \sin \theta \]

\[ |S| = \frac{P}{\cos \theta} = \frac{8000}{0.8} = 10 \text{ kVA} \]

\[ Q = 10 \sin \theta = 10 (0.6) = 6 \text{ kVAR} \]

\[ S = 0.8 + j 6 \text{ kVAR} \]

b) Load impedance

\[ P_{ave} = \frac{V_m I_m \cos \theta}{2} \]

\[ V_{rms} \cdot I_{rms} \cdot \cos \theta \]

8 kW = (240) \( I_{rms} \cdot 0.8 \)
\[ J_{\text{rms}} = \frac{8000}{240 \times 0.8} = 41.67 \, A \]

\[ \theta = \theta_0 - \phi = 36.87^\circ \]

\[ Z = \frac{V}{I} = \frac{|V|}{|I|} e^{j(\theta - \phi)} = \frac{|V|}{|I|} e^{j(\theta - \phi)} \]

\[ = \frac{V_{\text{m}}}{I_{\text{m}}} e^{j(36.87^\circ)} = \frac{V_{\text{rms}}}{I_{\text{rms}}} e^{j(36.87^\circ)} \]

\[ Z = 4.60 + j 3.45 \, \Omega \]

10.5 POWER CALCULATIONS

\[ S = \frac{V_{\text{m}} I_{\text{m}}}{2} [\cos(\theta_0 - \phi i) + j \frac{V_{\text{m}} I_{\text{m}}}{2} \sin(\theta_0 - \phi i)] \]

\[ = \frac{V_{\text{m}} I_{\text{m}}}{2} e^{j(\theta_0 - \phi i)} = \frac{1}{2} V_{\text{rms}} I_{\text{rms}} L_{\theta_0 - \phi i} \quad (10.27) \]

Equivalently, using effective values,

\[ S = V_{\text{eff}} I_{\text{eff}} L_{\theta_0 - \phi i} \quad (10.28) \]
(10.28) \[ S = V_{rms} I_{rms} e^{j\theta} = V_{rms} e^{j\theta} I_{rms} e^{-j\theta}, \]

\[ S = \frac{V_{rms} I_{rms}}{2} \]

\[ S = \frac{V I^*}{2} \]

Application: Example 10.1 (p. 10.10)

\[ v = 100 \cos (\omega t + 15^\circ) V = 100/15^\circ V \]

\[ i = 4 \, \sin (\omega t - 15^\circ) A = 4/105^\circ A \]

\[ \text{Get: } \begin{align*}
\text{Re } Q &= 173.21 \, \text{VA} \\text{Im } Q &= -100 \, \text{VA}
\end{align*} \]

\[ Q = 100/120^\circ \]

\[ Q = -100 + j 173.21 \, \text{VA} \]

\[ Q = \frac{P}{\text{Re } Q} = 173.21 \, \text{VAR} \]
Alternative forms of Complex Power

So:

\[ S = \frac{V_{\text{eff}} J_{\text{eff}}}{2} \quad (10.29) \]

First variation

\[ V_{\text{eff}} = Z J_{\text{eff}} \quad (10.31) \]

10.31 \rightarrow 10.29 gives

\[ S = Z \frac{V_{\text{eff}} J_{\text{eff}}}{2} = |J_{\text{eff}}|^2 Z \]

\[ = |J_{\text{eff}}|^2 (R + jX) \]

\[ = |J_{\text{eff}}|^2 R + j |J_{\text{eff}}|^2 X \]

\[ = P + j Q \]

where \( R \) - resistance of circuit

\( X \) - reactance of circuit (could be + or -)

Second variation

\[ S = \frac{V_{\text{eff}} J_{\text{eff}}}{2} = \frac{V_{\text{eff}} J_{\text{eff}}}{|V_{\text{eff}}|^2} = \frac{V_{\text{eff}} (V_{\text{eff}})^*}{2} \]
\[ S = \frac{N_{eff}^2}{Z^*} = P + jQ \quad \text{(1.935)} \]

Note: If \( Z \) purely resistive, \( P = \frac{|V_{eff}|^2}{R} \)

If \( Z \) purely reactive, \( Q = \frac{|V_{eff}|^2}{X} \)

where \( X \) positive or negative

Example 10.5  Calculating Average Reactive Power

\[ V_L = (39 + j26)(4 - j3) = 234.36 \angle -31.87^\circ \, \text{V (rms)} \]

\[ I_L = \frac{250 V}{40 + j30} = 4 - j3 = 5 \angle -31.87^\circ \, \text{A (rms)} \]

b) \( (P_{ave})_{\text{Load}} = |I_{\text{eff}}|^2 R = (25)^2 \times 39 = 975 \quad \text{(Watt-Joule energy)} \)

\( (Q)_{\text{Load}} = |I_{\text{eff}}|^2 \times 26 = (25)(26) = 650 \)
or equivalently,

\[ S = V_L I_L^* = (234 - j13)(4 + j3) = 975 + j650 \text{ VA} \]

c) Average reactive power delivered to the line:

\[ (\text{Part})_{\text{Line}} = |I_{\text{eff}}|^2 R_{\text{Line}} = (25)^1 = 25 \text{ mW} \]

\[ (\text{R} \text{e} \text{a} \text{c} \text{t} \text{i} \text{v} \text{e})_{\text{Line}} = |I_{\text{eff}}|^2 X = (25)^4 = 100 \text{ vars} \]

d) Average reactive power absorbed by source:

\[ (\text{Part})_{\text{Source}} \]

Average reactive power absorbed =

\[ (\text{Average reactive power absorbed to load}) \]

\[ (\text{" + " + " + " + " Line}) \]

\[ S_{\text{source}} = |I_{\text{eff}}|^2 Z = |I_{\text{eff}}|^2 \left[ (R_{\text{Line}} + jX_{\text{Line}}) + (R_{\text{Load}} + jX_{\text{Load}}) \right] \]

\[ = |I_{\text{eff}}|^2 (R_{\text{Line}} + jX_{\text{Line}}) \]

\[ + |I_{\text{eff}}|^2 (R_{\text{Load}} + jX_{\text{Load}}) \]

\[ = S_{\text{Line}} + S_{\text{Load}} \]

\[ = (25 + j100) + (975 + j650) = 1000 + j750 \text{ VA} \]

\[ : S_{\text{Source}} = -1000 - j750 \text{ VA} \]
**Example 10.6**

\[
I_s = 0.05, \qquad 30^\circ
\]

Load 1 absorbs \( P_r = 8 \text{ kvar} \), leading \( \text{PF} = 0.8 \) \( \cos \theta = 0.8 \quad \theta = 36.87^\circ \)

Load 2 absorbs \( |I| = 20 \text{ kVA} \), leading \( \text{PF} = 0.6 \) \( \Rightarrow \theta = 53.13^\circ \)

a) **Determine PF of 2 Loads**

Recall: \( S = P + j Q \)

\[
\begin{align*}
P &= \frac{1}{2} Vm I_m \cos \theta + j \frac{1}{2} Vm I_m \sin \theta = \frac{1}{2} V I^* \\
Q &= \frac{1}{2} Vm I_m \sin \theta - j \frac{1}{2} Vm I_m \cos \theta = -\frac{1}{2} V I^*
\end{align*}
\]

And:

\[
\begin{align*}
S_1 &= V I_1^* = Z_1 I_1^* = Z_1 |I_1|^2 \quad \text{(assume rms)} \\
S_2 &= V I_2^* = Z_2 I_2^* = Z_2 |I_2|^2 \\
S_{TOTAL} &= V (I_1^* + I_2^*) = V I_1^* + V I_2^* = S_1 + S_2
\end{align*}
\]

Load 1:

\[
8000 = V_{rms} I_{rms} (0.8) = 250 \cdot I_{rms} \cdot 0.8 \quad \Rightarrow \quad I_{rms} = \frac{8000}{250 \times 0.8}
\]

\[
|I_1| = I_{rms} e^{j36.87^\circ}
\]

Also can find \( I_2 \):

\[
I_s = I_1 + I_2
\]

Can find difference in phase between \( V \) and \( I_s \).
Alternatively.

\[ S_1 = 8000 - j6000 \text{ VA} \]

\[ S = 20000 + j10000 \text{ VA} \]

\[ \cos \theta = 0.8944 \text{ lagging} \]

\[ S_2 = 12000 + j16000 \text{ VA} \]

b) i) Apparent power to supply the loads:

\[ |S| = \sqrt{P^2 + Q^2} = \sqrt{(20000)^2 + (10000)^2} = 22360 \text{ KVA} \]

\[ S = \sqrt{S^*} \]

\[ I_s = \frac{20000 + j10000}{250} = 80 + j40 \text{ A} \]

\[ |I_s| = \sqrt{80^2 + 40^2} = 89.44 \text{ A} \]

iii) \[ |I_s| = 80 - j40 \text{ A} \]

\[ |I_s| = \sqrt{80^2 + 40^2} = 89.44 \text{ A} \]

c) SKIP
10.6 Maximum Power Transfer

\[ Z_L = ? \] for maximum \[ \text{Pare} \]

transfer to terminals \[ a, b \].

Can model any external circuit by a

Thyristor equivalent

\[ Z_{TH} = R_{TH} + jX_{TH} \]

\[ Z_L = R_L + jX_L \]

Note: Reactance can be transposed.

Also assume \( V_{TH} \) in rms value.

as reference phaser.

\[ \hat{I} = \frac{\hat{V}_{TH}}{(R_{TH} + R_L) + j(X_{TH} + X_L)} \]

\[ (\text{Pare}) \text{delivered to } R_L = |\hat{I}|^2 R_L \]

\[ P = \frac{|\hat{V}_{TH}|^2}{(R_{TH} + R_L)^2 + (X_{TH} + X_L)^2} \cdot R_L \]
\[ \frac{\partial P}{\partial X_L} = 0 \quad \frac{\partial P}{\partial R_L} = 0 \]

\[ X_L = -X_{TH} \quad R_L = \sqrt{R_{TH}^2 + (X_L + X_{TH})^2} \]

\[ Z_L = R_L + jX_L = R_{TH} + jX_{TH} = Z_{TH}^* \]

Max. Power absorbed (delivered to Load)

\[ Z_L = Z_{TH}^* \]

\[ I_{rms} = \frac{V_{TH}}{R_{TH} + jX_{TH} + R_L + jX_L} = \frac{V_{TH}}{2R_L} \]

\[ \max (P)_{rms} = \frac{1}{2} I_{rms}^2 R_L = \frac{1}{4} \frac{|V_{TH}|^2}{R_L} \]

\[ R_L = \frac{1}{4} \frac{|V_{TH}|^2}{R_L} \]
Max Power Transfer when $Z$ is restricted

For max power transfer when $Z_L = Z_{TH}$

(i) when this not possible, then optimum values of $R_c$, $X_L$

is the adjust $X_L$ as near as $-X_{TH}$ as possible as

$R_c = \frac{1}{\sqrt{1 + (X_L + X_{TH})^2}}$

(ii) adjust $|Z_L| = |Z_{TH}|$

Ass. Problem 10.7

a) $Z_L = ?$

For max power transfer, set $R_c = 0$

$Z_{TH} = 4 + j18 + \frac{-j300}{20 - j40} = 20 + j10 = 22.36/26.57^\circ$

$V_{TH} = 3 \frac{-180}{20 - j40} = 48 - j24 = 53.67/26.57^\circ V$

For max power transfer $Z_L = (20 - j10) \Omega$
b) \[ I = \frac{53.67 \angle -26.57^\circ}{40} = 1.34 \angle -26.57^\circ \]

\[ P_{\text{ac}} = \left| I \right|^2 R = \left( \frac{1.34}{\sqrt{2}} \right)^2 \cdot 20 = 17.96 \text{ W} \]

c) \[ R_L = \left| Z_{TH} \right| = 22.36 \Omega \]

d) \[ I = \frac{53.67 \angle -26.57^\circ}{42.36 + j10} = 1.23 \angle -39.83^\circ \text{ A} \]

\[ P_{\text{ac}} = \left( \frac{1.23}{\sqrt{2}} \right)^2 \cdot (22.36) = 17 \text{ W} \]