11.5 Power calculations in balanced 3-phase circuits.

Balanced Y load.

Active power associated with a-phase

\[ P_A = |V_{AN}| |I_{AN}| \cos (\theta_{VA} - \theta_{IA}) \]
\[ P_B = |V_{BN}| |I_{BN}| \cos (\theta_{VB} - \theta_{IB}) \]
\[ P_C = |V_{CN}| |I_{CN}| \cos (\theta_{VC} - \theta_{IC}) \]

In balanced 3-phase, magnitude of each line to neutral voltage is the same, as is magnitude of each phase current.

\[ V_\phi = |V_{AN}| = |V_{BN}| = |V_{CN}| \]
\[ I_\phi = |I_{AN}| = |I_{BN}| = |I_{CN}| \]

\[ \phi_\phi = \theta_{VA} - \theta_{IA} = \theta_{VB} - \theta_{IB} = \theta_{VC} - \theta_{IC} \]
Also, for balanced system, power delivered to each phase of the load is the same:

\[ P_A = P_B = P_C = P_\Phi = V_\Phi I_\Phi \cos \theta_\Phi \]

Total average power \[ P_T = 3 P_\Phi = 3 V_\Phi I_\Phi \cos \theta_\Phi \]

Using line voltages \[ V_L, I_L \] magnitudes of line voltages and currents:

\[ P_T = 3 \frac{V_L}{\sqrt{3}} I_L \cos \theta_\Phi \]

\[ = \sqrt{3} V_L I_L \cos \theta_\Phi \]

where we recognize that \[ |I_L| = |I_{inA1}| \ etc . \]

And \( \theta \), magnitude of line current = magnitude of phase currents. \( \Phi \Phi \) phase angle between phase voltage and phase current.
Complex Power in Balanced Y Load.

Active Power

\[ Q_\phi = V_\phi I_\phi \sin \phi \]

\[ Q_T = 3 Q_\phi = \sqrt{3} V_L I_L \sin \phi \]

For balanced load

\[ S_\phi = V_{AN} I_{aA}^* = V_{BN} I_{bB}^* = V_{CN} I_{cC}^* = V_\phi I_\phi^* \]

\[ S_T = 3 S_\phi = \sqrt{3} V_L I_L \angle \phi \]

Thus,

\[ S_\phi = P_\phi \text{Re} + Q_\phi \text{Im} = V_\phi I_\phi^* \]

\[ S_T = 3 S_\phi = \sqrt{3} V_L I_L \angle \phi \]

11-17
c) magnitude of line voltage at sending end of line

\[ V_{an} = \sqrt{0.005^2 + 0.025^2} \left( 577.36 \angle -36.47^\circ \right) + \frac{600}{\sqrt{3}} \]

\[ = 357.51 \angle 1.57^\circ \text{ V} \]

Thus, \[ V_L = \sqrt{3} | V_{an} | = 619.23 \text{ V} \]

d) Power factor at sending end of line

\[ \cos (\theta - \theta_0) = \cos (1.57 - (-36.47)) = \cos 38.44^\circ \]

\[ = 0.783 \text{ kVAR} \]
Example 11.1

Terminology:
- Phase voltages at generator terminals: $V_a$, $V_b$, $V_c$, $V_n$
- Phase voltages at load terminals (magnitudes): $V_{a_n}$, $V_{b_n}$, $V_{c_n}$ (in magnitude)
- Line voltages at generator branches: $V_{a_b}$, $V_{b_c}$, $V_{c_a}$
- Line voltages at load branches: $V_{a_b}$, $V_{b_c}$, $V_{c_a}$

Graph:

\[ V_{ab} = I_{aA} Z_{ea} + V_{ab} - I_{bB} Z_{eb} = V_{AB} \]
Example 11.5

Balanced 3-phase
Line impedance: \(0.005 + j0.025\ \Omega\)

\[\text{Example: \(P_f = 480 \text{ kW}\)}
\]

\[\text{Phase \(P_f = 0.8\)}
\]

\[\text{Line voltage: 660 V @ 0.8 \text{ rms}}\]

(a) Single-phase equivalent circuit

\[\text{a)}\]

\[\text{b)}\]

Line current magnitude

\[\text{Power associated with a phase:} \frac{480 \text{ kW}}{3} = 160 \text{ kVA}\]

\[\text{So:} \ 160 \text{ kVA} = V_{\text{rms}} I_{\text{rms}} \cos \theta_f\]

\[\text{Calculate the current:} \frac{600}{\sqrt{3}} I_{\text{rms}} 0.8\]

\[V_{\text{rms}} I_{\text{rms}} = \frac{160,000 \sqrt{3}}{110 (1.8)} = 577.35\]

Also:

\[S = \sqrt{3} (160 + j120) \times 10^3 = \frac{600}{\sqrt{3}} I_1\]

\[I_1 = \frac{\sqrt{3} (160 + j120) \times 10^3}{600} \text{ (\text{rms})}\]

\[\text{Magnitude:} \sqrt{3} \sqrt{\frac{600^2 + 120^2}{600^2}} \times 0.8\]

\[\text{(I_{AA}) :} I_1 = 577.35 \times 36.87\]

\[\geq 577.35\]