**Instructions**: Present your work in a neat and organized manner. Please use either the 8.5 × 11 size paper or the filler paper with pre-punched holes. Please do not use paper which has been torn from a spiral notebook. Please secure all your papers by using either a staple or a paper clip, but not by folding its (upper left) corner.

You must show all of the essential details of your work to get full credit. If I am forced to fill in gaps in your solution by using notrivial (at my discretion) steps, I will reduce your score for that particular assignment.

Please refer to the syllabus for the instructions on working on homework assignments with other students and on submitting your own work.

All problems in a given assignment contribute equal amount (1 point) to the assignment’s total score, unless otherwise noted.

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**Homework Assignment # 9**

**Due Wednesday, November 6, 2013**


   *Note*: Do not use the formula from Question A to answer Question B. In fact, the only place in this assignment where you may use the answer to Question A is Problem 2(i).


   *Hint*: To begin, write this Möbius transformation as a composition of a translation, a rotation, and then another translation. The parameters of these transformations are yet undetermined; you task is precisely to determine them. To that end, use two observations. First, the angle of the rotation should be evident from the information given. Second, consider the point that must be fixed for this Möbius transformation. This gives you a relation connecting the parameters of the two translations. Now you can finish this problem.

   **Additional assignment**: Look at your answer and state what the first translation achieves and then what the second translation does relative to the first. Also, try to relate this to the role played by $z = 1$ in this problem.

2. (0.5 point)


   *Hint*: You may use a formula from Problem 1(i).


3. (1.5 points)

   (i) M. Henle, *Modern geometries*, Chap. 5, p. 63: # 3(a,b).

   The Hint in the book should have referred to page 58, not 60.

   *Technical note*: You will arrive at an equation of the form

   $$(w - A)/(w - B) = \lambda (z - C)/(z - D),$$

   which you will need to solve for $w$. First, denote the entire r.h.s. by, say, $X$ and then solve for $w$ in terms of $X$. Second, substitute the expression for $X$ (i.e., $X = \lambda (z - C)/(z - D)$) and simplify your answer.

   *Note for # 3(a)*: When taking the limit, refer to Problem 1(iii)(b,c) in HW 8.
(ii) In Lec. 14 we studied inversion. More specifically, it was inversion about $z = 0$. In that case the center of inversion, $z = 0$, is transformed to $w = \infty$, and vice versa.

Now, find a Möbius transformation which is the inversion about point $z = 1$. (That is, $z = 1$ is the center of inversion for this Möbius transformation.)

**Hint:** There are more than one method to solve this problem (you only need to do it by one method). For example, one can identify two easy-to-guess points whose images under this Möbius transformation are also easy to find. Then, to be able to use the technique from part (i) of this problem, you need to find the third point and its image. For that, note that the inversion in question should transform any point on the unit circle centered at $z = 1$ into a point on the same circle that is symmetric to the original point with respect to the real axis. So, express the fact stated in the previous sentence as a mathematical formula and use it to complete your calculation. After you obtain an answer as a fraction, put it in a final form by doing long division as in Problem 1(i).

Another method is to use the idea of Problem 1(ii). I.e., represent the required Möbius transformation as a composition of three simpler transformations.

(iii) After you have obtained an answer for part (ii) (regardless of which of the two methods you used), you can identify its similarity with that in Problem 1(ii). Based on the observed similarity only and without doing any calculations, write down a formula for a homothetic transformation about point $z = 1$.

4. **(0.5 point)** M. Henle, *Modern geometries*, Chap. 5, p. 64: # 6 (skip the third of the given transformations).

5. M. Henle, *Modern geometries*, Chap. 5, p. 64: # 17. For each pair of plots, you must explain what Möbius transformation takes one of the plots into its image.

**Hint:** Follow the lines of a similar example in Lec. 15. Namely, assume that some point in the original picture is sent to infinity and then see what happens to the rest of the picture. You should expect to make a few (incorrect) attempts before guessing the correct point that needs to be sent to infinity.

6. **(0.5 point)** L.-s. Hahn, *Complex numbers and geometry*, Chap. 3, p. 160: # 13(a,b,c).

**Note:** Despite this being a simple problem (notice that three problems combined are worth only half a point), its results will be extensively used for justification of certain constructions in hyperbolic geometry.

**Hint for # 13(a,b):** All you need to do here is to refer to one or more of the appropriate basic constructions. The radius and center of circle $C$ are assumed to be given (as usual).

**Hint for # 13(c):** Use your result from # 13(a) or (b), whichever applies. For simplicity, out of many possible relative locations of the circles and $P$, consider the one where the circles do not intersect and are not one inside the other, and also where $P$ is outside of both circles.

**Bonus** (Credit for each part will be given only if the solution for this part is more than 50% correct.)


**Hint for # 15(a):** Even though $C$ is arbitrary, you may first transform it to the unit circle. Explain why such a transformation will not change your answer. Then, the rest of the problem is a straightforward calculation.