ERRATA

Abstract Algebra, Third Edition

by D. Dummit and R. Foote

(most recently revised on February 14, 2018)

These are errata for the Third Edition of the book. Errata from previous editions have been fixed in this edition so users of this edition do not need to refer to errata files for the Second Edition (on this web site). Individuals using the Second Edition, however, must make corrections from this list as well as those in the Second Edition errata files (except for corrections to text only needed in the Third Edition; for such text no reference to Second Edition page numbers is given below). Some of these corrections have already been incorporated into recent printings of the Third Edition.

page vi (2^{nd} Edition p. vi)

from: 7.3 Ring Homomorphisms an Quotient Rings
to: 7.3 Ring Homomorphisms and Quotient Rings

page 2, Proposition 1(1) (2^{nd} Edition p. 2, Proposition 1(1))

from: The map f is injective
to: If A is not empty, the map f is injective

page 4, line -3 (2^{nd} Edition p. 5, line 3)

from: $a, b \in \mathbb{Z} - \{0\}$
to: $a, b \in \mathbb{Z}$ and $b \neq 0$

page 8, line -9 (2^{nd} Edition p. 8, line -9)

from: For any $k \in \mathbb{Z}$
to: For any $a \in \mathbb{Z}$

page 31, The group S_3 table

last line missing
add: $\sigma_6(1) = 3$, $\sigma_6(2) = 1$, $\sigma_6(3) = 2$ | $(1\ 3\ 2)$

page 33, Exercise 10, line 2 (2^{nd} Edition p. 33, Exercise 10)

from: its least residue mod m when $k + i > m$
to: its least positive residue mod m

page 34, line 1 of Definition (2^{nd} Edition p. 34, line 1 of Definition)

from: two binary operations
to: two commutative binary operations

page 39, Example 2, line -4

from: $ba = ab^{-1}$
to: $ba = a^{-1}b$

page 44, Exercise 7 (2^{nd} Edition p. 44, Exercise 7)

from: the action is faithful.
to: the action is faithful when the vector space is nonzero.

page 45, Exercise 22 (2$^{\text{nd}}$ Edition p. 46, Exercise 22)
from: is isomorphic to a subgroup (cf. Exercise 26 of Section 1) of S_4
to: is isomorphic to S_4

page 51, line −1 (2$^{\text{nd}}$ Edition p. 52, line −1)
from: see Exercise 1 in Section 1.7
to: see Exercise 4(b) in Section 1.7

page 65, line 2 of Exercise 16(c) (2$^{\text{nd}}$ Edition p. 66, Exercise 16(c))
from: if and only H
to: if and only if H

page 71, Exercise 5 (2$^{\text{nd}}$ Edition p. 72, Exercise 5)
from: there are 16 such elements x
to: there are 8 such elements x

page 84, line 11 of Example 2 (2$^{\text{nd}}$ Edition p. 85, line 11 of Example 2)
from: By Proposition 2.6
to: By Theorem 2.7(1)

page 84, line −6 of Example 2 (2$^{\text{nd}}$ Edition p. 85, line −6 of Example 2)
from: By Proposition 2.5
to: By Theorem 2.7(3)

page 86, Exercise 14(d) (2$^{\text{nd}}$ Edition p. 87, Exercise 14(d))
from: root
to: roots

page 98, Figure 6
add: hatch marks to upper right and lower left lines of the central diamond (to indicate $AB/B \cong A/A \cap B$).

page 103, line 3 of Definition (2$^{\text{nd}}$ Edition p. 104, line 3 of Definition)
from: N_{i+1}/N_i a simple group
to: N_{i+1}/N_i is a simple group

page 114, line 3 in proof of Proposition 2 (2$^{\text{nd}}$ Edition p. 116, line 3 of proof)
from: $b \in G$
to: $g \in G$

page 117, Exercise 10(a) (2$^{\text{nd}}$ Edition p. 119, Exercise 10(a))
from: cosets $x_1K, \dots, x_nK$ where $\{x_1K, \dots, x_nK\}$
to: cosets x_iK where $\{x_iK \mid i \in I\}$

page 128, third line above last display (2$^{\text{nd}}$ Edition p. 130, line -4)
from: cycle type (2,2)
to: cycle type 1,2,2

page 128, second line above last display (2nd Edition p. 130, line -3)

from: any element of odd order

to: any nonidentity element of odd order

page 131, Exercise 19 (2nd Edition p. 133, Exercise 19)

from: Prove that $\mathcal{K}$ is a union ... $k = |G : HC_G(x)|$.

to: Prove that $\mathcal{K}$ is a union of conjugacy classes of H of equal size, and the number of these classes is $|G : HC_G(x)|$.

page 132, Exercise 33, line -1 (2nd Edition p. 134, line -1 of Exercise 33)

from: See Exercises 6 and 7 in Section 1.3

to: See Exercises 16 and 17 in Section 1.3

page 132, Exercise 36(c) (2nd Edition p. 135, Exercise 36(c))

from: g and h lie in the center of G

to: g and h lie in the center of G and $g = h^{-1}$

page 139, Definition (1) (2nd Edition p. 141, Definition (1))

from: A group of order p^α for some $\alpha \geq 1$

to: A group of order p^α for some $\alpha \geq 0$

page 143, last line of first Example (2nd Edition p. 145, line -2)

from: Theorem 17

to: Proposition 17

page 145, line -7 (2nd Edition p. 148, line 5)

from: less that

to: less than

page 148, Exercise 41 (2nd Edition p. 150, Exercise 41)

from: existing wording

to: Prove that $SL_2(\mathbb{F}_4) \cong A_5$, where $\mathbb{F}_4 = \{0, 1, a, a+1\}$ is a field of order 4 (cf. Exercise 1, Section 2.1 and Exercise 1, Section 3.3).

page 148, Exercise 47(i) (2nd Edition p. 151, Exercise 47(i))

from: that has some prime divisor p such that n_p is not forced to be 1

to: for each prime divisor p of n the corresponding n_p is not forced to be 1

page 149, Exercise 53 (2nd Edition p. 151, Exercise 53)

from: G is any non-abelian group

to: G is any non-abelian finite group

page 149, Exercise 54, line 4 (2nd Edition p. 151, line 4 of Exercise 54)

from: G/N acts as automorphisms of N

to: $G/C_G(N)$ acts as automorphisms of N

page 151, Exercise 6, line -2 (2nd Edition p. 153, line -2 of Exercise 6)

from: every pair of elements of D lie in a finite simple subgroup of D

to: every pair of elements of A lie in a finite simple subgroup of A

page 156, Exercise 8, line 2 (2nd Edition p. 158, Exercise 8)
from: that the map $\pi \mapsto \varphi_\pi$ is an injective homomorphism
to: that when $G_1 \neq 1$, the map $\pi \mapsto \varphi_\pi$ is an injective homomorphism

page 158, line 3 after the Definition (2nd Edition p. 160)
from: n-tuple
to: r-tuple

page 161, Theorem 5(3) (2nd Edition p. 163, Theorem 5(3))
from: if $G \cong B_1 \times B_2 \times \cdots \times B_m$
to: if $G \cong B_1 \times B_2 \times \cdots \times B_k$

page 164, step (4) (2nd Edition p. 166, step (4))
from: each of the t (ordered) lists
to: each of the k (ordered) lists

page 174, Exercise 18 (2nd Edition p. 176, Exercise 18)
from: Let $K_1, K_2, \ldots, K_n$ be etc.
to: Let $G_1, G_2, \ldots, G_n$ be etc. (change K_i to G_i throughout the exercise).

page 180, line -4 (2nd Edition p. 182, line -4)
from: some proper normal subgroup
to: some nontrivial proper normal subgroup

page 187, Exercise 23, line 4 (2nd Edition p. 189, line 4 of Exercise 23)
from: from G into
to: from K into

page 187, Exercise 25 (2nd Edition p. 189, Exercise 25)
from: and that $H(\mathbb{F}_p)$ has exponent p
to: and that for p odd, $H(\mathbb{F}_p)$ has exponent p

page 191, Proposition 2 (2nd Edition p. 193, Proposition 2)
from: nilpotence class at most $a-1$.
to: nilpotence class at most $a-1$ for $a \geq 2$ (and class equal to a when $a = 0$ or 1).

page 191, line 3 of the proof of Proposition 2 (2nd Edition p. 193)
from: Thus if $Z_i(P) \neq G$
to: Thus if $Z_i(P) \neq P$

page 194, Theorem 8, line 4 (2nd Edition p. 196, Theorem 8, line 4)
from: $Z_i(G) \leq G^{c-i-1} \leq Z_{i+1}(G) \quad \text{for all } i \in \{0, 1, \ldots, c-1\}$.
to: $G^{c-i} \leq Z_i(G) \quad \text{for all } i \in \{0, 1, \ldots, c\}$.

page 198, Exercise 18 (2nd Edition p. 200, Exercise 18)
from: then $G'' = 1$
to: then $G'' = G'''$

page 199, Exercise 22 (2^{nd} Edition p. 201, Exercise 22)
from: Prove that
to: When G is a finite group prove that

page 201, line 2 of Exercise 38 (2^{nd} Edition p. 203, Exercise 38)
from: The group G/M
to: The group P/M

page 209, Proposition 14(1)
from: $n_3 = 7$
to: $n_3 = 28$

page 216, line 4 after displayed steps (1) and (2) (2^{nd} Edition p. 217, line -3)
from: are equal if and only if $n = m$ and $\delta_i = \epsilon_i$, $1 \le i \le n$
to: are equal if and only if $n = m$, $r_i = s_i$ and $\delta_i = \epsilon_i$, $1 \le i \le n$

page 217, line 2 after first display (2^{nd} Edition p. 218, line 2 after second display)
from: $A(F)$ be the subgroup
to: $A(S)$ be the subgroup

page 219, line 4 of Example 1 (2^{nd} Edition p. 220, line 4 of Example 1)
from: $R_0 \le \ker \pi$
to: $\langle R_0 \rangle \le \ker \pi$

page 219, line 11 of Example 1 (2^{nd} Edition p. 220, line 11 of Example 1)
from: S must be a generating set for G, and
to: S must be a generating set for G satisfying the relations in R, and

page 249, line 2 of Exercise 17(a) (2^{nd} Edition p. 250, Exercise 17(a))
from: every ring homomorphism from R to S
to: every nonzero ring homomorphism from R to S

page 255, line 2 of Example 1 (2^{nd} Edition p. 256, line 2 of Example 1)
from: We saw in Section 3
to: We saw in Section 1

page 260, Exercise 40(iii) (2^{nd} Edition p. 261, Exercise 40(iii))
from: $R/\eta(R)$
to: $R/\mathfrak{N}(R)$

page 263, line 2 of the Definition (2^{nd} Edition p. 264)
from: ring of fractions of D with respect to R
to: ring of fractions of R with respect to D

page 264, Exercise 2 (2^{nd} Edition p. 265, Exercise 2)
from: let D be a nonempty subset of R that is closed under multiplication
to: let D be a nonempty subset of R that does not contain 0 and is closed under multiplication

page 269, line 2 of Exercise 10(c) (2^{nd} Edition p. 270, Exercise 10(c))
from: then A may likewise
to: then P may likewise

page 269, line 2 of Exercise 11(d) (2nd Edition p. 270, Exercise 11(d))
from: Prove that every ideal of
to: Prove that every nonzero ideal of

page 269, lines 1 and 2 of Exercise 11(e) (2nd Edition p. 270, Exercise 11(e))
from: in the direct limit $\mathbb{Z}_p$ satisfying a_j^p
to: in the inverse limit $\mathbb{Z}_p$ satisfying a_j^{p-1}

page 274, first paragraph after Definition, and Proposition 2 (2nd Edition p. 275)
from: ring R
to: ring R with 1

page 282, second display (2nd Edition p. 283, second display)
from: $0 < N(\frac{\alpha}{\beta}s - t) = \dfrac{(ay - 19bx - cq)^2 + 19(ax + by + cz)^2}{c^2} \leq \dfrac{1}{4} + \dfrac{19}{c^2}$
and so $(*)$ is satisfied with this s and t provided $c \geq 5$.
to: $0 < N(\frac{\alpha}{\beta}s - t) = \dfrac{(ay - 19bx - cq)^2 + 19(ax + by + cz)^2}{c^2} = \dfrac{r^2 + 19}{c^2} \leq \dfrac{1}{4} + \dfrac{19}{c^2}$
and so $(*)$ is satisfied with this s and t provided $c \geq 5$ (note $r^2 + 19 \leq 23$ when $c = 5$).

page 283, Exercise 8 (2nd Edition p. 284, Exercise 8)
from: D is a multiplicatively closed subset of R
to: D is a multiplicatively closed subset of R with $0 \notin D$

page 290, line 5 (2nd Edition p. 291, line 5)
from: is irreducible in $\mathbb{Z}[i]$
to: is irreducible in $\mathcal{O}$

page 292, line 4 of Example
from: $(4 + i)(5 - 2i) = 22 - 3i$, $(4 - i)(5 - 2i) = 22 + 3i$
to: $(4 + i)(5 - 2i) = 22 - 3i$, $(4 - i)(5 + 2i) = 22 + 3i$

page 293, Exercise 5(b) [Hint] (2nd Edition p. 293, Exercise 5(b))
from: [Show that either $\sqrt{-n}$ or $1 + \sqrt{-n}$ is not prime.]
to: [Show that 2 is not prime.]

page 303, last line (2nd Edition p. 304, last line)
from: Multiplying through by a common denominator
to: Multiplying through by the product of all denominators

page 304, line 13 (2nd Edition p. 305, line 3)
from: one fewer irreducible factors
to: one fewer irreducible factor

page 306, line 3 (2nd Edition p. 307, line3)
from: where $p'(x)$ is irreducible in both $R[x]$ and $F[x]$.
to: where $p'(x)$ is irreducible in $R[x]$ if and only if it is irreducible in $F[x]$.

page 312, Exercise 16(b) (2nd Edition p. 313, Exercise 16(b))
from: Prove that f
to: If $f(0) \neq 0$, prove that f

page 316, line –6

from: Suppose next that $d < N$. In this case $a \in L_d$ for some $d < N$,

to: Thus $a \in L_d$ for some $d < N$,

page 316, line –5

from: $\cdots + r_{n_d} b_{n_d}$ for some $\ldots$ $+ r_{n_d} f_{n_d}$ is a

to: $\cdots + r_{n_d} b_{d,n_d}$ for some $\ldots$ $+ r_{n_d} f_{d,n_d}$ is a

page 318, line 7 after the Definition

from: $L(fg) = L(f) + L(g)$

to: $L(fg) = L(f)L(g)$

page 323, line -6

from: among the differences $S(g_i, g_j)$

to: among the remainders of the differences $S(g_i, g_j)$

page 330, line 7

Replace from "We close this section ..." to "Example" with:

We close this section by showing how to compute the basic set-theoretic operations of sums, products and intersections of ideals in polynomial rings. Suppose $I = (f_1, \ldots, f_m)$ and $J = (h_1, \ldots, h_k)$ are two ideals in $F[x_1, \ldots, x_n]$. Then $I + J = (f_1, \ldots, f_m, h_1, \ldots, h_k)$ and $IJ = (f_1 h_1, \ldots, f_i h_j, \ldots, f_m h_k)$. The following proposition shows how to compute the intersection of any two ideals.

Proposition 30. Suppose $I = (f_1, \ldots, f_m)$ and $J = (h_1, \ldots, h_k)$ are two ideals in $F[x_1, \ldots, x_n]$. If $\mathcal{I}$ denotes the ideal generated by $tf_1, \ldots, tf_m, (1-t)h_1, \ldots, (1-t)h_k$ in $F[t, x_1, \ldots, x_n]$, then $I \cap J = \mathcal{I} \cap F[x_1, \ldots, x_n]$. In particular, $I \cap J$ is the first elimination ideal of $\mathcal{I}$ with respect to the ordering $t > x_1 > \cdots > x_n$.

Proof: If $f \in I \cap J$, then $f = tf + (1-t)f$, and noting both tf and $(1-t)f$ are in $\mathcal{I}$ shows $I \cap J \subseteq \mathcal{I} \cap F[x_1, \ldots, x_n]$. Conversely, suppose $f \in \mathcal{I} \cap F[x_1, \ldots, x_n]$. Then $f = a_1 t f_1 + \cdots + a_m t f_m + b_1(1-t)h_1 + \cdots + b_k(1-t)h_k$ for some polynomials $a_1, \ldots, a_m, b_1, \ldots, b_k$ in $F[t, x_1, \ldots, x_n]$. Setting $t = 0$ (which does not alter f) shows f is an $F[x_1, \ldots, x_n]$-linear combination of $h_1, \ldots, h_k$, so $f \in J$. Similarly, setting $t = 1$ shows $f \in I$, so $f \in I \cap J$. Finally, since $I \cap J = \mathcal{I} \cap F[x_1, \ldots, x_n]$, $I \cap J$ is the first elimination ideal of $\mathcal{I}$ with respect to the ordering $t > x_1 > \cdots > x_n$.

By Propositions 29 and 30, if $I = (f_1, \ldots, f_m)$ and $J = (h_1, \ldots, h_k)$, then the elements not involving t in a Gröbner basis for the ideal generated by $tf_1, \ldots, tf_m$ and $(1-t)h_1, \ldots, (1-t)h_k$ in $F[t, x_1, \ldots, x_n]$, computed for the lexicographic monomial ordering $t > x_1 > \cdots > x_n$, give a Gröbner basis for the ideal $I \cap J$ in $F[x_1, \ldots, x_n]$.

page 331, Exercise 3(i)

from: minimum element

to: minimal element

page 332, Exercise 9(b)

from: grlex order

to: grevlex order

page 332, Exercise 15(a)

from: Prove that $\{g_1, \dots, g_m\}$ is a minimal Gröbner basis for the ideal I in R if

to: Prove that the subset $\{g_1, \dots, g_m\}$ of the ideal I in R is a minimal Gröbner basis of I if

page 332, Exercise 16, line 3

from: $(LT(g_1), \dots, LT(g_m), LT(S(g_i, g_j))$ is strictly larger than the ideal $(LT(g_1), \dots, LT(g_m))$. Conclude that the algorithm ...

to: $(LT(g_1), \dots, LT(g_m), LT(r))$ is strictly larger than the ideal $(LT(g_1), \dots, LT(g_m))$, where $S(g_i, g_j) \equiv r \bmod G$. Deduce that the algorithm ...

page 333, display in Definition following Exercise 33

from: $rJ \in I$

to: $rJ \subseteq I$

page 334, Exercise 43(a)

from: Use Exercise 30

to: Use Exercise 39

page 334, Exercise 43(b)

from: Use Exercise 33(a)

to: Use Exercise 42(a)

page 334, line 3 of Exercise 43(c)

from: ideal defined in Exercise 32,

to: ideal quotient (cf. Exercise 41),

page 343, line 5 of Example 4 (2^{nd} Edition p. 324, Example 4)

from: Exercise 23

to: Exercise 22

page 348, line 6 (2^{nd} Edition p. 329, line 6)

from: When R is a field, however

to: When R is a field and $M \neq 0$, however

page 350, line 2 of Exercise 4 (2^{nd} Edition p. 331, Exercise 4)

from: $\varphi(\overline{k}) = ka$

to: $\varphi_a(\overline{k}) = ka$

page 357, line 1 of Exercise 17 (2^{nd} Edition p. 338, Exercise 17)

from: assume further that the ideals

to: assume further that R is commutative and the ideals

page 357, Exercise 21(i) (2^{nd} Edition p. 338, Exercise 21(i))

replace (i) with: the map from the (external) direct sum $\oplus_{i \in I} N_i$ to the submodule of M generated by all the N_i's by sending a tuple to the sum of its components is an isomorphism (cf. Exercise 20)

page 360, line 6 of second paragraph (2nd Edition p. 340, line -5)

Remove the second comma in: i.e., ,

page 368, line 4 of Proof (2nd Edition p. 349, line 4 of proof)

from: $\Phi : M \otimes_R N$

to: $\Phi : M \otimes_R N \to L$

page 370, line 16 of Example 8 (2nd Edition p. 351, line 4)

from: $f(n \bmod I)$

to: $f(n \bmod IN)$

page 372, Corollary 16(2), top line of commutative diagram

from: $M \times \cdots \times M_n \xrightarrow{\iota} M \otimes \cdots \otimes M_n$

to: $M_1 \times \cdots \times M_n \xrightarrow{\iota} M_1 \otimes \cdots \otimes M_n$

page 374, line 2 of second Remark (2nd Edition p. 355 line 2)

from: Section 11.6

to: Section 11.5

page 374, lines -1, -3, -4 (2nd Edition p. 355, lines 13, 14, 16)

from: f (four occurrences)

to: φ

page 375, line 4 of Exercise 8 (2nd Edition p. 356, Exercise 8)

from: relation $(u, n) \sim (u', n)$ if and only if $u'n = un'$ in N.

to: relation $(u, n) \sim (u', n')$ if and only if $xu'n = xun'$ in N for some $x \in U$.

page 377, Exercise 23 (2nd Edition p. 357, Exercise 23)

from: Proposition 19

to: Proposition 21

page 377, Exercise 25 (2nd Edition p. 358, Exercise 25)

from: Let R be a subring of the commutative ring S

to: Let S be a commutative ring containing R (with $1_S = 1_R$)

page 385, title of subsection following Proposition 26

from: Modules and $\mathrm{Hom}_R(D, \_\_)$

to: Projective Modules and $\mathrm{Hom}_R(D, \_\_)$

page 394, line 10 of second paragraph (2nd Edition p. 374, line 26)

from: $id_N \circ \psi \circ \varphi = 0$, i.e., $\psi \circ \varphi = 0$,

to: $id_N \circ \varphi \circ \psi = 0$, i.e., $\varphi \circ \psi = 0$,

page 395, line 7 after the Definition (2nd Edition p. 376, line 4)

from: Put another way, the map $\mathrm{Hom}_R(D, \_\_)$

to: Put another way, the map $\mathrm{Hom}_R(\_\_, D)$

page 396, line –2 above Proposition 36 (2nd Edition p. 376)

from: Exercises 18 and 19

to: Exercises 19 and 20

page 398, proof of Theorem 38 (2nd Edition p. 378)

from: Exercises 15 to 17
to: Exercises 15 and 16

page 399, line 8 (2nd Edition p. 379, line 22)

from: The map $1 \otimes \varphi$ is not in general injective
to: The map $1 \otimes \psi$ is not in general injective

page 399, line -9 (2nd Edition p. 380, line 3)

from: $\widetilde{\pi} : D \times N \to$
to: $\widetilde{\pi} : D \otimes N \to$

page 401, line 2 of Example 1 (2nd Edition p. 381, line 2 of Example 1)

from: $\mathbb{Z}/2\mathbb{Z}$ not a flat module
to: $\mathbb{Z}/2\mathbb{Z}$ is not a flat module

page 402, line 7 (2nd Edition p. 382, line 7 of Proof)

from: mapping (a, b) to $\Phi(a)(c)$
to: mapping (a, b) to $\Phi(a)(b)$

page 403, Exercise 1(d) (2nd Edition p. 383, Exercise 1(d))

from: if β is injective, α and γ are surjective, then γ is injective
to: if β is injective, α and φ are surjective, then γ is injective

page 405, Exercise 15 (2nd Edition p. 385, Exercise 115)

change exercise to:

Let M be a left $\mathbb{Z}$-module and let R be a ring with 1.
(a) Show that $\mathrm{Hom}_{\mathbb{Z}}(R, M)$ is a left R-module under the action $(r\varphi)(r') = \varphi(r'r)$ (see Exercise 10).

(b) Suppose that $0 \to A \xrightarrow{\psi} B$ is an exact sequence of R-modules. Prove that if every $\mathbb{Z}$-module homomorphism f from A to M lifts to a $\mathbb{Z}$-module homomorphism F from B to M with $f = F \circ \psi$, then every R-module homomorphism f' from A to $\mathrm{Hom}_{\mathbb{Z}}(R, M)$ lifts to an R-module homomorphism F' from B to $\mathrm{Hom}_{\mathbb{Z}}(R, M)$ with $f' = F' \circ \psi$. [Given f', show that $f(a) = f'(a)(1_R)$ defines a $\mathbb{Z}$-module homomorphism of A to M. If F is the associated lift of f to B, show that $F'(b)(r) = F(rb)$ defines an R-module homomorphism from B to $\mathrm{Hom}_{\mathbb{Z}}(R, M)$ that lifts f'.]
(c) Prove that if Q is an injective $\mathbb{Z}$-module then $\mathrm{Hom}_{\mathbb{Z}}(R, Q)$ is an injective R-module.

page 407, last line of Exercise 27(a) (2nd Edition p. 387, Exercise 27(a))

from: where π_1 and π_2 are the natural projections onto
to: where π_1 and π_2 are the restrictions to X of the natural projections from $A \oplus B$ onto

page 409, line 2 of Definition (1) (2nd Edition p. 389, Definition (1))

from: and $v_1, v_2, \ldots, v_n \in S$
to: and $v_1, v_2, \ldots, v_n$ distinct elements of S

page 410, line -6 (2nd Edition p. 390)

Replace from display (11.2) to end of page with:

$$\gamma_1 b_1 + \cdots + \gamma_k b_k + \gamma_{k+1} b_{k+1} + \gamma_{k+2} a_{k+2} + \cdots + \gamma_n a_n = 0 \tag{11.2}$$

then substituting for b_{k+1} from the expression for b_{k+1} in equation (1), we obtain a linear combination of $\{b_1, b_2, \ldots, b_k, a_{k+1}, a_{k+2}, \ldots, a_n\}$ equal to 0, where the coefficient of a_{k+1} is $\gamma_{k+1}\alpha_{k+1}$. Since this last set is a basis by induction, all the coefficients in this linear combination must be 0, and so $\gamma_{k+1} = 0$ since $\alpha_{k+1} \neq 0$. But then equation (2) is

$$\gamma_1 b_1 + \cdots + \gamma_k b_k + \gamma_{k+2} a_{k+2} + \cdots + \gamma_n a_n = 0.$$

page 423, line 3 of Exercise 9 (2^nd Edition p. 403, Exercise 9)

from: If $\varphi|_W$ and $\widetilde{\varphi}$ are

to: If $\varphi|_W$ and $\bar{\varphi}$ are

page 426, line 2 of Exercise 21(b) (2^nd Edition p. 406, Exercise 21(b))

from: $6z$

to: $+6z$

page 433, proof of Theorem 19, line 3

from: $= E_v(f) + \alpha E_g(v)$

to: $= E_v(f) + \alpha E_v(g)$

page 435, Exercise 1

change exercise to:

Let V be a vector space over F of dimension $n < \infty$. Prove that the map $\varphi \mapsto \varphi^*$ in Theorem 20 is a vector space isomorphism of $\mathrm{End}(V)$ with $\mathrm{End}(V^*)$, but is not a ring homomorphism when $n > 1$. Exhibit an F-algebra isomorphism from $\mathrm{End}(V)$ to $\mathrm{End}(V^*)$.

page 442, line −8 (2^nd Edition p. 422, line −8)

from: $\varphi : M \to A$ is an R-algebra

to: $\varphi : M \to A$ is an R-module

page 445, second display in Theorem 34(2) (2^nd Edition p. 425, Theorem 34(2))

from: $\iota(m_1, \ldots, m_k) = m_1 \otimes \cdots \otimes m_n \bmod \mathcal{C}(M)$

to: $\iota(m_1, \ldots, m_k) = m_1 \otimes \cdots \otimes m_k \bmod \mathcal{C}^k(M)$

page 446, lines 11 and 12 of Proof (2^nd Edition p. 426, lines 11 and 12 of Proof)

from: where v_i^a denotes ... taken a times

to: where $v_i^{a_i}$ denotes ... taken a_i times

page 459, line −8 (2^nd Edition p. 439,line −8)

from: $y_i = a_{1i}e_i + a_{2i}e_2 + \cdots + a_{ni}e_i$

to: $y_i = a_{1i}e_1 + a_{2i}e_2 + \cdots + a_{ni}e_n$

page 468, Exercise 1(a) (2^nd Edition p. 448, Exercise 1(a))

from: Change first two sentences to

to: Show that any finite number of elements of M, one of which is torsion, are R-linearly dependent.

page 469, line 1 of Exercise 10 (2^nd Edition p. 449, Exercise 10)

from: N an R-module

to: N a torsion R-module

page 479, last sentence of second paragraph (2^nd Edition p. 459, second paragraph)

from: the degree of the minimal polynomial for A has degree at most n

to: the minimal polynomial for A has degree at most n

page 490, Exercise 21 (2^{nd} Edition p. 470, Exercise 21)
from: Prove that ... multiplies the determinant by a unit.
to: Prove that a second elementary row and column operation described before Theorem 21 does not change the determinant of the matrix and the first and third elementary operations multiply the determinant by a unit.

page 510, line 1 of text (2^{nd} Edition p. 490, line 1 of text)
from: F is a commutative ring with
to: F is a nonzero commutative ring with

page 516, line 3 or Remark (2^{nd} Edition p. 496, line 3 of Remark)
from: examples indicates
to: examples indicate

page 526, lines 1 and 2 (2^{nd} Edition p. 505, last paragraph lines 1 and 2)
from: the algebraic α is obtained by adjoining the element α to F
to: the algebraic element α is obtained by adjoining α to F

page 552, lines 4 below section head and last display (2^{nd} Edition p. 532, ditto)
from: $1 \leq a < n$
to: $1 \leq a \leq n$

page 552, line -5 (2^{nd} Edition p. 532, line -5)
from: which is also a d^{th} root of unity
to: that is also a primitive d^{th} root of unity

page 555, Exercise 7, bounds for product (2^{nd} Edition p. 535, Exercise 7)
from: $d \mid n$
to: $d \mid m$

page 562, line -4 (2^{nd} Edition p. 542, line -4)
from: any polynomial over $\mathbb{Q}$
to: any polynomial $f(x)$ over $\mathbb{Q}$

page 566, Example 7, first line after second display
from: we see that $\sigma_p^{p^n} = 1$
to: we see that $\sigma_p^n = 1$

page 579, line -11 (2^{nd} Edition p. 559, line -11)
from: minimal polynomial $\Phi_4(x)$
to: minimal polynomial $\Phi_8(x)$

page 582, Exercise 17 (2^{nd} Edition p. 563, Exercise 17)
from: Let K/F be any finite extension
to: Let K/F be any finite separable extension

page 584, Exercise 24 (2nd Edition p. 564, Exercise 24)
change exercise to:

Prove that the rational solutions $a, b \in \mathbb{Q}$ of Pythagoras' equation $a^2 + b^2 = 1$ are of the form $a = \dfrac{s^2 - t^2}{s^2 + t^2}$ and $b = \dfrac{2st}{s^2 + t^2}$ for some $s, t \in \mathbb{Q}$. Deduce that any right triangle with integer sides has sides of lengths $((m^2 - n^2)d,\ 2mnd,\ (m^2 + n^2)d)$ for some integers m, n, d. [Note that $a^2 + b^2 = 1$ is equivalent to $\mathrm{N}_{\mathbb{Q}(i)/\mathbb{Q}}(a + ib) = 1$, then use Hilbert's Theorem 90 above with $\beta = s + it$.]

page 585, Exercise 29(b) (2nd Edition p. 565, Exercise 29(b))
from: Prove that the element $t =$
to: Prove that the element $s =$

page 585, Exercise 29(c) (2nd Edition p. 565, Exercise 29(c))
from: Prove that $k(t)$
to: Prove that $k(s)$

page 587, Proposition 18 (2nd Edition p. 567, Proposition 18)
from: all the distinct irreducible polynomials
to: all the distinct monic irreducible polynomials

page 593, line 7 (2nd Edition p. 573, line 7 of proof of Proposition 21)
from: the squarefree part of the polynomial $f_1(x)f_2(x)$
to: the least common multiple in $F[x]$ of $f_1(x)$ and $f_2(x)$

page 597, line 1 of Example 2
from: $\mathbb{Q}(\zeta_{13})$,For p
to: For p

page 617, Exercises (2nd Edition p. 598, Exercises)

The first 10 exercises, excluding Exercise 3, are over the field $\mathbb{Q}$.

page 638, Exercise 18 (2nd Edition p. 619, Exercise 18)
from: Let $D \in \mathbb{Z}$ be a squarefree integer
to: Let $D \neq 1$ be a squarefree integer

page 644, $\mathbf{A_7}$ line of Table (2nd Edition p. 625)
from: 21 (second column: for cycles of type 2 entry)
to: blank (no 2-cycles in A_7)

page 653, Exercise 9(b), displayed diagram (2nd Edition p. 634, Exercise 9(b))
change: All E to $\mathbb{F}_3$, and the bottom field to $\mathbb{F}_3\left(\dfrac{(t^6 + t^4 + t^2 + 1)^2}{(t^3 - t)^3}\right)$

page 654, Exercise 16 (2nd Edition p. 635, Exercise 16)
from: Prove that F does not contain all quadratic extensions of $\mathbb{Q}$.
to: Prove that F does contain all quadratic extensions of $\mathbb{Q}$. [One way is to consider the polynomials $x^3 + 3ax + 2a$ for $a \in \mathbb{Z}^+$.]

page 670, line 2 of Exercise 34 (2^{nd} Edition p. 648, Exercise 34)

from: $\mathrm{Ass}_R(N) \subseteq \mathrm{Ass}_R(M)$

to: $\mathrm{Ass}_R(L) \subseteq \mathrm{Ass}_R(M)$

page 673, line 2 of Section 15.2 (2^{nd} Edition p. 650)

from: zero locus

to: zero loci

page 679, line 12

from: $\mathbb{R}[x, y, z, t]$

to: $\mathbb{R}[x, y, t]$

page 687, Exercise 13 (2^{nd} Edition p. 662, Exercise 13)

change exercise to:

Let V be a nonempty affine algebraic set. Prove that if $k[V]$ is the direct sum of two nonzero ideals then V is not connected in the Zariski topology. Prove the converse if k is algebraically closed. [Use Theorem 31.] Give a counterexample to the converse when k is not algebraically closed.

page 705, Exercise 18(a)

change exercise part (a) to:

Show that I and J are radical ideals that are not prime. Conclude that $I = \mathcal{I}(V)$ and $J = \mathcal{I}(W)$ and that V and W are reducible algebraic sets.

page 707, line 2 of Corollary 37(1) (2^{nd} Edition p. 678, Corollary 29(1))

from: if and only if D contains no zero divisors of R

to: if and only if D contains no zero divisors or zero

page 713, line 7 of Example 1

from: $P_2 \cap \mathbb{Q}[y, z] = (y^5 - z^4)$

to: $P_2 = P \cap \mathbb{Q}[y, z] = (y^5 - z^4)$

page 721, line 4 after commutative diagram (2^{nd} Edition p. 688, line 2)

from: By Proposition 38(1) [$\mathbf{2^{nd}}$ Edition: By Proposition 30(1)]

to: By Proposition 46(1) [$\mathbf{2^{nd}}$ Edition: By Proposition 36(1)]

page 726, first line of Exercises (2^{nd} Edition p. 693)

from: D is a multiplicatively closed set in R.

to: D is a multiplicatively closed set in R with $1 \in D$.

page 728, Exercise 21, line 1

from: Suppose $\varphi : R \to S$ is a ring homomorphism

to: Suppose $\varphi : R \to S$ is a ring homomorphism with $\varphi(1_R) = 1_S$

page 732, first display (2^{nd} Edition p. 698)

from: $\mathcal{Z}(A) = \{P \in X \mid A \subseteq P\} \subseteq \mathrm{Spec}R$,

to: $\mathcal{Z}(A) = \{P \in \mathrm{Spec}R \mid A \subseteq P\}$,

page 741, line 8 (2^{nd} Edition p. 707, line 12)

from: P, where

to: v, where

page 754, line 2 of Exercise 8 (2^{nd} Edition p. 720, Exercise 8)
from: Observe the
to: Observe that

page 756, line 1 of proof of Proposition 5 (2^{nd} Edition p. 722, proof of Proposition 5)
from: $\nu(u) + \nu(v) = \nu(uv) = 1$
to: $\nu(u) + \nu(v) = \nu(uv) = \nu(1) = 0$

page 761, paragraph after Definition (2^{nd} Edition p. 727)
replace paragraph with:
If R is a P.I.D. then the class number is 1. The converse is true if every ideal of R is invertible (a family of such rings is studied in the next section), but is not true in general; it is an exercise to show that the class number of a Bezout Domain is trivial, but a Bezout Domain need not be a P.I.D. (see Exercises 12 in Section 9.2, 5 in Section 9.3, and 23 in Section 16.3 for examples).

page 761, line 3 of proof of Proposition 10 (2^{nd} Edition pp. 727)
from: $g : A \to F$ by $f(c) =$
to: $g : A \to F$ by $g(c) =$

page 764, line -2 (2^{nd} Edition p. 730, line -2)
from: Every Principal Ideal Domain is
to: Every Principal Ideal Domain that is not a field is

page 767, line 5 (2^{nd} Edition pp. 733)
from: complete
to: completes

page 774, line 2 of Exercise 12
from: in R are relatively prime
to: in R that are relatively prime

page 775, lines 1 to 3 of Exercise 24(d) (2^{nd} Edition pp. 741–2, Exercise 24(d))
from: $P_3 = (3, 1 + \sqrt{-5}) = (3, 5 - \sqrt{-5})$ ...... [Check that $\sqrt{-10} = -(5 - \sqrt{-5})\omega/3$.]
to: $P_3 = (3, 1 - \sqrt{-5}) = (3, 5 + \sqrt{-5})$ ...... [Check that $\sqrt{-10} = (5 + \sqrt{-5})\omega/3$.]

page 781, bottom row of diagram (17.9) (2^{nd} Edition p. 748, diagram (17.9))
from: $0 \longrightarrow \mathrm{Hom}_R(A, D) \longrightarrow$
to: $0 \longrightarrow \mathrm{Hom}_R(A', D) \longrightarrow$

page 793, line 4 of Exercise 11(c) (2^{nd} Edition p. 760, Exercise 11(c))
from: projection maps $I \to I_i$
to: projection maps $I \to I/I_i$

page 794, Exercise 17 (2^{nd} Edition p. 761, Exercise 17)
from: for any abelian group A
to: for any abelian group B

page 799, line 2 after (17.17) (2^{nd} Edition p. 765, line 2 after (17.17))
from: in Theorem 8
to: in Theorem 10

page 800, line -7 (2^{nd} Edition p. 766, line -7)
from: $H^n(G,A) \cong Ext^n(\mathbb{Z},A)$
to: $H^n(G,A) \cong Ext^n_{\mathbb{Z}G}(\mathbb{Z},A)$

page 801, line 4 (2^{nd} Edition p. 767, line 4)
from: 1 if n is odd
to: a if n is odd

page 810, line 3 of Exercise 2(c) (2^{nd} Edition p. 776, Exercise 2(c))
from: $\psi : P_n \to F_n$
to: $\psi : F_n \to P_n$

page 812, Exercise 18(a) (2^{nd} Edition p. 778, Exercise 18(a))
from: from $\mathbb{Z}/(m/d)\mathbb{Z}$ to $\mathbb{Z}/m\mathbb{Z}$ if n is odd, and from 0 to 0 if n is even, $n \geq 2$,
to: from 0 to 0 if n is odd, and from $\mathbb{Z}/(m/d)\mathbb{Z}$ to $\mathbb{Z}/m\mathbb{Z}$ if n is even, $n \geq 2$,

page 813, line 3 of Exercise 19 (2^{nd} Edition p. 779, Exercise 19)
from: p-primary component of $H^1(G,A)$
to: p-primary component of $H^n(G,A)$

page 815, line 2 of Proposition 30 (2^{nd} Edition p. 781)
from: group homomorphisms from G to H
to: group homomorphisms from G to A

page 816, line -13 (2^{nd} Edition p. 782, line -13)
from: bijection between the elements of
to: bijection between the cyclic subgroups of order dividing n of

page 823, Exercise 9(b) (2^{nd} Edition p. 789, Exercise 9(b))
from: $H^1(A_n,V) = 0$ for all p
to: $|H^1(A_n,V)| = \begin{cases} 3, & \text{if } p = 3 \text{ with } n = 4 \text{ or } 5 \\ 0, & \text{otherwise} \end{cases}$

page 832, lines -10 and -14 (2^{nd} Edition p. 798, lines -6 and -10)
from: L
to: K

page 853, line 3 of Exercise 14(c) (2^{nd} Edition p. 819, Exercise 14(c))
from: all $A \in H$
to: all $A \in H$ and all $g \in G$

page 853, line 4 of Exercise 17 (2^{nd} Edition p. 819, Exercise 17)
from: Your proof ...
to: Your proof that φ has degree 1 should also work for infinite abelian groups when φ has finite degree.

page 857, line 7 (2nd Edition p. 823, line 7)
from: *Proof of Proposition 6* Let
to: *Proof of Proposition 6:* Let

page 869, line −6 (2nd Edition p. 835, line −6)
from: the isotypic components of G
to: the isotypic components of M

page 885, Exercise 8 (2nd Edition p. 851, Exercise 8)
from: This table contains nonreal entries.
to: This table contains irrational entries.

page 893, line 4 (2nd Edition p. 859, line 4)
from: a proper, nontrivial subgroup of G
to: a proper, nontrivial normal subgroup of G

page 897, line 7(2nd Edition p. 863)
Replace from "Let $\psi \ldots$" to end of proof with:

Let $\mathcal{C}$ be the set of nonprincipal irreducible characters of Q. For each $\psi \in \mathcal{C}$ and each $i = 0, 1, \ldots, p-1$ define

$$\psi_i(h) = \psi(x^i h x^{-i}) \qquad \text{for all } h \in Q.$$

Since ψ_i is a homomorphism from Q into $\mathbb{C}^\times$ it is also an irreducible character of Q. Thus $P = \langle\, x \,\rangle$ permutes $\mathcal{C}$ via the (right) action $\psi^{x^i} = \psi_i$ (see Exercise 10).

If $\psi_i = \psi_j$ for some $i > j$ then $\psi(x^i h x^{-i}) = \psi(x^j h x^{-j})$ and so $\psi(h) = \psi(x^{i-j} h x^{j-i})$ for all $h \in Q$. Let $k = i - j$ so that $\psi = \psi_k$. Thus $\ker\psi = \ker\psi_k$ and it follows that x^k normalizes $\ker\psi$. Since $\langle\, x \,\rangle = \langle\, x^k \,\rangle$ acts irreducibly on Q, $\ker\psi = 1$. Thus ψ is a faithful character. But G is a Frobenius group so $h \neq x^k h x^{-k}$ for every nonidentity $h \in Q$, contrary to $\psi(h) = \psi(x^k h x^{-k})$. This proves $\psi_0, \ldots, \psi_{p-1}$ are distinct irreducible characters of Q, i.e., P acts without fixed points on $\mathcal{C}$.

Next let $\psi \in \mathcal{C}$ and let $\Psi = \mathrm{Ind}_Q^G(\psi)$. We use the orthogonality relations and the preceding results to show that Ψ is irreducible. Since $1, x^{-1}, \ldots, x^{-(p-1)}$ are coset representatives for Q in G and, by Corollary 12, Ψ is zero on $G - Q$ we have

$$\begin{aligned}
||\,\Psi\,||^2 &= \frac{1}{|G|} \sum_{h \in Q} \Psi(h)\overline{\Psi(h)} \\
&= \frac{1}{|G|} \sum_{h \in Q} \sum_{i=0}^{p-1} \psi(x^i h x^{-i}) \sum_{j=0}^{p-1} \overline{\psi(x^j h x^{-j})} \\
&= \frac{1}{|G|} \sum_{i,j=0}^{p-1} \sum_{h \in Q} \psi_i(h)\overline{\psi_j(h)} \\
&= \frac{1}{|G|} |Q| \sum_{i,j=0}^{p-1} (\psi_i, \psi_j)_Q = \frac{1}{|G|} |Q| p = 1
\end{aligned}$$

where the second line follows from the definition of the induced character Ψ, and the last line follows because the previous paragraph gives $(\psi_i, \psi_j)_Q = \delta_{ij}$. This proves Ψ is an irreducible character of G.

Finally we show that every irreducible character of G of degree > 1 is induced from some nonprincipal degree 1 character of Q by counting the number of distinct irreducible characters of G obtained this way. By parts (1) and (2) the number of irreducible characters of G (= the number of conjugacy classes) is $p + (q^a - 1)/p$ and the number of degree 1 characters is p. Thus the number of irreducible characters of G of degree > 1 is $(q^a - 1)/p$. Each $\psi \in \mathcal{C}$ induces to an irreducible character of degree p of G. Characters ψ_i, ψ_j in the same orbit of P acting on $\mathcal{C}$ induce to the same character of G (which is zero outside Q and on Q it is $\sum_{i=0}^{p-1} \psi_i$). One easily computes that characters in different orbits of P on $\mathcal{C}$ induce to orthogonal irreducible characters of G. Since P acts without fixed points on $\mathcal{C}$, the number of its orbits is $|\mathcal{C}|/p = (q^a - 1)/p$. This accounts for all irreducible characters of G of degree > 1, and all such have degree p. The proof is complete.

page 899, line 1 of item (3) (2^{nd} Edition p. 865)
from: let Q_3 be a Sylow 11-subgroup of G
to: let Q_3 be a Sylow 13-subgroup of G

page 904, line 2 of Exercise 10(c) (2^{nd} Edition p. 870, Exercise 10(c))
from: Prove also that if ψ_1 and ψ_2 are characters of N
to: Prove also that if ψ_1 and ψ_2 are irreducible characters of N

page 907, Exercise 1(a) (2^{nd} Edition p. 873, Exercise 1(a))
from: a 3-tuple in $A \times A \times A$ maps to an ordered pair in $A \times A$
to: an ordered pair in $A \times A$ maps to a 3-tuple in $A \times A \times A$

page 912, line 6 (2^{nd} Edition p. 878, line 6)
from: if $A \neq B$ or $C \neq D$
to: if $A \neq C$ or $B \neq D$