

EE-171 HW # 607 SOLUTIONS (contd.)

3.1, 3.3, 3.4, 3.5, 3.7

3.1) $T = 8$

$a_1 = a_{-1} = 2$

$a_3 = a_{-3} = 4j$

$$x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k)$$

$$x(t) = \hat{a}_1 e^{j(2\pi/T)t} + \hat{a}_{-1} e^{-j(2\pi/T)t} + \hat{a}_3 e^{j3(2\pi/T)t} + \hat{a}_{-3} e^{-j3(2\pi/T)t}$$

$$x(t) = 2 \left(e^{j(\pi/4)t} + e^{-j(\pi/4)t} \right) + 4j \left(e^{j3(\pi/8)t} - e^{-j3(\pi/8)t} \right)$$

$$x(t) = 4 \cos(\pi/4 t) - 8 \sin(6\pi/8 t)$$

3.3)

$$x(t) = 2 + \cos(2\pi/3 t) + 4 \sin(5\pi/3 t)$$

$$= 2 + \frac{1}{2} e^{j2\pi/3 t} + \frac{1}{2} e^{-j2\pi/3 t} - 2j e^{j5\pi/3 t} + 2j e^{-j5\pi/3 t}$$

$$= 2 + \frac{1}{2} e^{j2(2\pi/6)t} + \frac{1}{2} e^{-j2(2\pi/6)t} - 2j e^{j5(2\pi/6)t} + 2j e^{-j5(2\pi/6)t}$$

$a_0 = 2$

$a_1 = \frac{1}{2}$

$a_{-1} = \frac{1}{2}$

$a_5 = -2j$

$a_{-5} = 2j$

3.4)

$$x(t) = \begin{cases} 1.5 & 0 \leq t < 1 \\ -1.5 & 1 \leq t < 2 \end{cases}$$

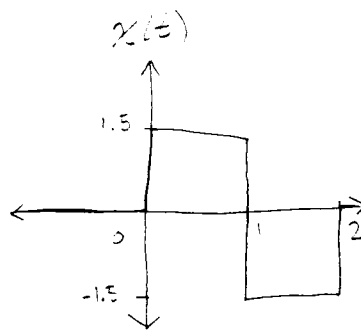
$\omega_0 = \pi$

$$a_k = \frac{1}{2} \int_0^2 x(t) e^{-jk\pi t} dt$$

$$a_0 = \frac{1}{2} \int_0^1 1.5 dt - \frac{1}{2} \int_1^2 1.5 dt$$

$$a_k = \frac{1}{2} \int_0^1 1.5 e^{-jk\pi t} dt - \frac{1}{2} \int_1^2 1.5 e^{-jk\pi t} dt$$

$$= \frac{3}{4} \left[\int_0^1 e^{-jk\pi t} dt - \int_1^2 e^{-jk\pi t} dt \right] = \frac{3}{4} \left(\frac{-1}{jk\pi} \left[e^{-jk\pi t} \right]_0^1 + \frac{1}{jk\pi} \left[e^{-jk\pi t} \right]_1^2 \right)$$



$$\hat{a}_k = \frac{-3}{4jk\pi} \left(-e^{-jk\pi} + 1 + e^{jk\pi} - e^{j2k\pi} \right)$$

$$= \frac{-3}{4jk\pi} \left(e^{-jk(\pi/2)} \left(e^{jk(\pi/2)} - e^{-jk(\pi/2)} \right) + e^{jk(\pi/2)} \left(-e^{-jk(\pi/2)} + e^{jk(\pi/2)} \right) \right)$$

$$\hat{a}_k = \frac{3e^{-jk(\pi/2)}}{2\pi} \sin(k\pi/2)$$

3.5) $x_2(t) = x_1(1-t) + x_1(t-1)$

$$T_2 = \frac{2\pi}{\omega_2} = T_1 = \frac{2\pi}{\omega_1}$$

$$x_1 \leftrightarrow a_k$$

$$x_1(t+1) \leftrightarrow a_k e^{j(2\pi/T_1)k}$$

$$x_1(t-1) \leftrightarrow a_k e^{-j(2\pi/T_1)k}$$

$$x_1(1-t) \leftrightarrow a_{-k} e^{-j(2\pi/T_1)k}$$

$$\therefore x_1(t+1) + x_1(1-t) \leftrightarrow a_k e^{j(2\pi/T_1)k} + a_{-k} e^{-j(2\pi/T_1)k} = e^{-j(2\pi/T_1)k} (a_k + a_{-k})$$

3.7)

$$\int_T^{2T} x(t) dt = 2 \quad x(t) \leftrightarrow a_k$$

$$g(t) = \frac{dx}{dt} \leftrightarrow b_k = jk \frac{2\pi}{T} a_k$$

$$a_k = \frac{b_k}{j(2\pi/T)k} \quad k \neq 0$$

$$a_0 = \frac{1}{T} \int_T^{2T} x(t) dt = \frac{2}{T}$$

$$a_k = \begin{cases} 2/T & k=0 \\ \frac{b_k}{j(2\pi/T)k} & k \neq 0 \end{cases}$$