EE 171

HW #5

2.45 a), b), c) 2.46 2.47 2.48

2.45) a) i)

$x(t) \rightarrow y(t)$

$x(t-h) \rightarrow y(t-h)$ BY TIME INVARIANCE PROPERTY

$x(t) - x(t-h) \rightarrow y(t) - y(t-h)$ BY LINEARITY PROPERTY AND TIME INVARIANCE PROPERTY

$\dfrac{x(t) - x(t-h)}{h} \rightarrow \dfrac{y(t) - y(t-h)}{h}$ BY LINEARITY PROPERTY AND TIME INVARIANCE PROPERTY

$$\lim_{h \to 0} \frac{x(t) - x(t-h)}{h} \rightarrow \frac{y(t) - y(t-h)}{h}$$

$\dfrac{dx}{dt} \rightarrow \dfrac{dy}{dt}$ BY DEFINITION OF THE DERIVATIVE

ii)

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(t-\tau) h(\tau) d\tau$$

$$y'(t) = \frac{dy}{dt} = \frac{d}{dt} \int_{-\infty}^{\infty} x(t-\tau) h(\tau) d\tau$$

$$y'(t) = \int_{-\infty}^{\infty} \frac{d}{dt}\left[x(t-\tau)\right] h(\tau) d\tau$$

$$= \int_{-\infty}^{\infty} x'(t-\tau) h(\tau) d\tau$$

$$y(t) = x'(t) h(t)$$

iii)

$$x(t) \longrightarrow \boxed{u_1(t)} \longrightarrow w(t) \longrightarrow \boxed{h(t)} \longrightarrow y'(t)$$

[FROM FIGURE P2.45]

$$x(t) * u_1(t) = w(t) = x'(t)$$

$$x(t) \longrightarrow \boxed{u_1(t)} \longrightarrow x'(t) \longrightarrow \boxed{h(t)} \longrightarrow y'(t)$$

$$y'(t) = x(t) * u_1(t) * h(t)$$

$$y'(t) = [x(t) * u_1(t)] * h(t)$$

$$y(t) = x'(t) * h(t)$$

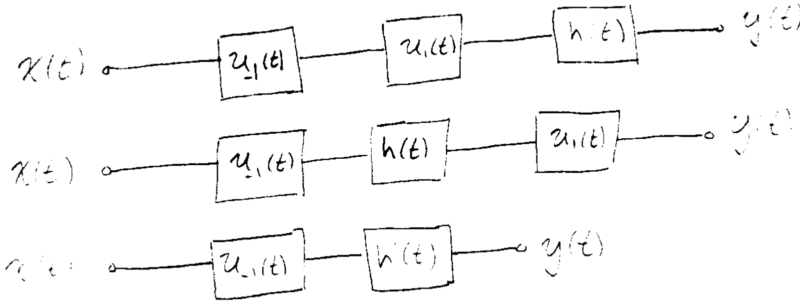
$$x'(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

2 i)

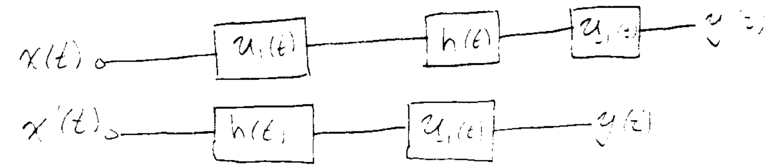
$$y(t) = x(t) * h(t)$$
$$= h(t) * x(t)$$
$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)\,x(\tau)\,d\tau$$
$$\frac{dy}{dt} = y'(t) = \frac{d}{dt}\int_{-\infty}^{\infty} h(t-\tau)\,x(\tau)\,d\tau$$
$$= \int_{-\infty}^{\infty} \frac{d}{dt} h(t-\tau)\,x(\tau)\,d\tau$$
$$= \int_{-\infty}^{\infty} \left[\frac{d}{dt} h(t-\tau)\right] x(\tau)\,d\tau$$
$$= \int_{-\infty}^{\infty} h'(t-\tau)\,x(\tau)\,d\tau$$
$$y'(t) = h'(t) * x(t)$$

ii) WE KNOW $u_1(t) * u_{-1}(t) = \delta(t)$

$$x(t) * u_{-1}(t) * u_1(t) * h(t) = y(t)$$
$$x(t) * u_{-1}(t) * h(t) * u_1(t) = y(t)$$
$$x(t) * u_{-1}(t) * [h(t) * u_1(t)] = y(t)$$
$$x(t) * u_{-1}(t) * h'(t) = y(t)$$



$$x(t) * u_{-1}(t) * h'(t) = y(t)$$
$$\left[\int_{-\infty}^{t} x(\tau)\,d\tau\right] * h'(t)$$



$$x(t) * u_1(t) * h(t) * u_{-1}(t) = y(t)$$
$$x'(t) * h(t) * u_{-1}(t) = y(t)$$
$$\int_{-\infty}^{t} x'(\tau)\,h(t-\tau)\,d\tau = y(t)$$

$x(t) * u_1(t) * h(t) * u(t) = y(t)$

$x(t) * h(t) * u(t) = y(t)$

$x(t) * \left(\int_{-\infty}^{\infty} h(\tau) u(t-\tau) d\tau \right) = y(t)$

$x(t) * \left(\int_{-\infty}^{t} h(\tau) d\tau \right) = y(t)$

c) $x(t) = e^{-5t} u(t)$

$y(t) = \sin \omega_0 t$

$$\frac{dx}{dt} = -5e^{-5t} u(t) + \delta(t) \rightarrow \frac{dy}{dt} = \omega_0 \cos \omega_0 t$$

$$-5(y(t)) + \delta(t) \longrightarrow \omega_0 \cos \omega_0 t$$

$$-5(\sin \omega_0 t) + \delta(t) \longrightarrow \omega_0 \cos \omega_0 t$$

$$\delta(t) \longrightarrow \omega_0 \cos \omega_0 t + 5 \sin \omega_0 t$$

$$\therefore h(t) = \omega_0 \cos \omega_0 t + 5 \sin \omega_0 t$$

2.46) $x(t) \rightarrow y(t)$

$x(t) = 2e^{-3t} u(t-1)$

$$\frac{dx}{dt} = -6e^{-3t} u(t-1) + 2\delta(t-1) \longrightarrow -3y(t) + e^{-2t} u(t)$$

$$-3(x(t)) + 2\delta(t-1) \longrightarrow -3y(t) + e^{-2t} u(t)$$

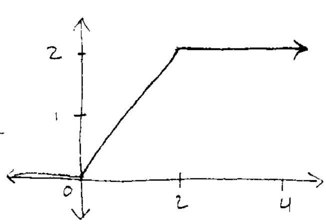
$$2\delta(t-1) \longrightarrow e^{-2t} u(t)$$

$$\delta(t-1) \longrightarrow \frac{1}{2} e^{-2t} u(t)$$

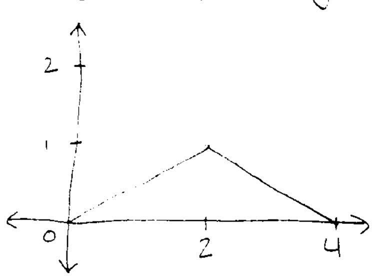
$$\delta(t) \longrightarrow \frac{1}{2} e^{-2(t+1)} u(t+1)$$

$$h(t) \rightarrow \frac{1}{2} e^{-2(t+1)} u(t+1)$$

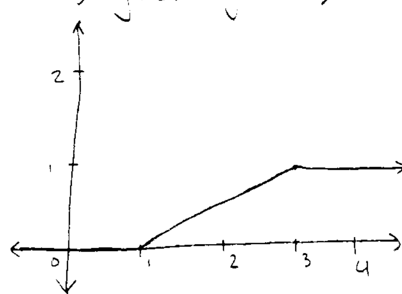
2.47) a) $y(t) = 2y_0(t)$



b) $y(t) = y_0(t) - y_0(t-2)$

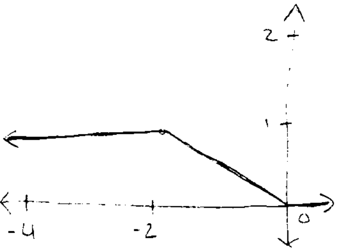


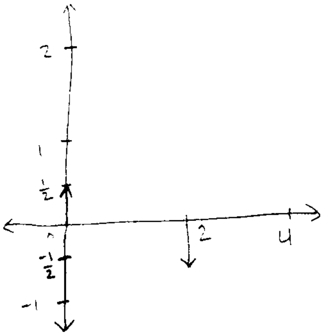
c) $y(t) = y_0(t-1)$



d) NOT ENOUGH INFORMATION

e) $y(t) = x(-t)$ f) $y(t) = \frac{d}{dt}x(t)$





2.48) a) TRUE. IF $h(t)$ IS PERIODIC THEN

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \infty$$

BECAUSE $h(t)$ IS <u>NOT</u> ABSOLUTELY INTEGRABLE IT IS <u>NOT</u> STABLE.

b) FALSE. SUPPOSE $h_1[n] = \delta[n-1]$ CAUSAL

INVERSE $\longrightarrow$ $h_2[n] = \delta[n+1]$ NON-CAUSAL

c) FALSE. SUPPOSE $h[n] = u[n]$. $|u[n]| \leq k$ WHEN $k=1$, HOWEVER

$$\sum_{n=-\infty}^{\infty} |u[n]| = \infty.$$

d) TRUE. SUPPOSE $h[n]$ IS NON-ZERO FOR $n_1 \leq n \leq n_2$,

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=n_1}^{n_2} |h[n]| < \infty$$

e) FALSE. SUPPOSE $h(t) = e^{t}u(t)$. $h(t)$ IS CAUSAL BUT NOT STABLE.

f) FALSE. $h_1[n] = \delta[n-1]$, $h_2[n] = \delta[n+1]$

$h_1[n] * h_2[n] = \delta[n]$

$h_1[n] \rightarrow$ CAUSAL

$h_2[n] \rightarrow$ NOT CAUSAL

g) FALSE. $h(t) = e^{-t}u(t)$

$s(t) = (1-e^{-t})u(t)$

$$\int_0^{\infty} |1-e^{-t}|\,dt = t + e^{-t}\Big|_0^{\infty} = \infty$$

ALTHOUGH $h(t)$ IS ABSOLUTELY INTEGRABLE, $s(t)$ IS <u>NOT</u>

h) TRUE. $u[n] = \sum_{k=0}^{\infty} \delta[n-k]$

$s[n] = \sum_{k=0}^{\infty} h[n-k]$

$\therefore$ $s[n] = 0$ FOR $n<0$ THEN $h[n] = 0$ FOR $n<0$ AND THE SYSTEM IS CAUSAL