HW # 11

9.3, 9.4, 9.5, 9.9, 9.10, 9.18, 9.19

9.3, 9.4, 9-5, 9-9

9-11 (Algrd from His)

3) $x(t) = e^{-5t}u(t) + e^{-\beta t}u(t)$

$$X(s) = \int_{-\infty}^{\infty} e^{-5t}u(t)e^{-st}\,dt + \int_{-\infty}^{\infty} e^{-\beta t}u(t)e^{-st}\,dt$$

$$= \int_{0}^{\infty} e^{(-5-s)t}\,dt + \int_{0}^{\infty} e^{(-\beta-s)t}\,dt$$

$$= \frac{1}{-5-s}\left[e^{(-5-s)t}\right]_0^{\infty} + \frac{1}{-\beta-s}\left[e^{(-\beta-s)t}\right]_0^{\infty}$$

$$= \frac{1}{5+s} + \frac{1}{\beta+s}$$

$\mathrm{Re}\{s\} > -3$ FROM THE PROBLEM DESCRIPTION

ROC: $\mathrm{Re}\{s\} > \max(-5, \mathrm{Re}\{\beta\})$

$\therefore \mathrm{Re}\{\beta\} = 3$, NO RESTRICTION ON $\mathrm{Im}\{\beta\}$

4) $$x(t) = \begin{cases} e^{t}\sin 2t & t \le 0 \\ 0 & t > 0 \end{cases}$$

FROM TABLE 9.2

$$e^{-t}\sin 2t\,u(t) \longleftrightarrow \frac{2}{(s+1)^2 + 2^2}$$

FROM TABLE 9.1

$$x(-t) = \frac{1}{|-1|}X(-s) = X(-s)$$

$$\therefore X(s) = \frac{2}{(s-1)^2 + 2^2}$$

FINDING POLES:

$(s-1)^2 + 4 = 0$

$s^2 - 2s + 5 = 0$

$$s = \frac{2 \pm \sqrt{4 - 4(1)(5)}}{2(1)} = \frac{2 \pm \sqrt{-16}}{2} = 1 \pm \sqrt{-4} = \boxed{1 \pm i2}$$

$\boxed{\text{ROC: } \mathrm{Re}(s) < 1}$

5) a) $$\frac{1}{s+1} + \frac{1}{s+3} = \frac{(s+1) + (s+3)}{(s+1)(s+3)} = \frac{2s+4}{(s+1)(s+3)}$$

$2s + 4 = 0$

$\underline{s = -2}$

$$= \frac{2s+4}{s^2+4s+3}$$

BECAUSE THE DENOMINATOR GROWS FASTER THAN THE NUMERATOR, IT ALSO HAS A ZERO AT...

$\underline{s = \infty}$

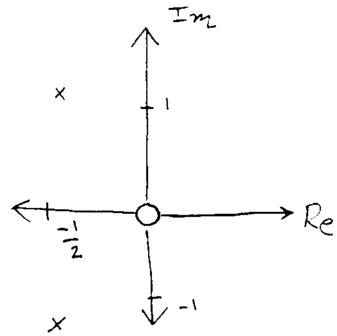
1 ZERO IN THE FINITE PLANE,
1 ZERO AT INFINITY.

b)

$$H_2(s) = \frac{s}{s^2 + s + 1}$$

ZEROS: $s = 0$

POLES: $s = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)} = \frac{-1}{2} \pm \frac{i\sqrt{3}}{2}$



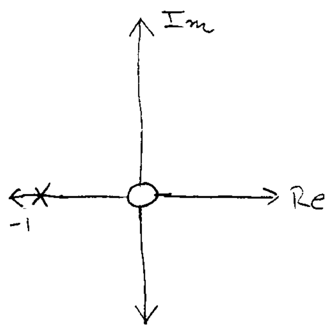
$\boxed{\text{BANDPASS}}$

c) $$H_3(s) = \frac{s^2}{s^2 + 2s + 1}$$

ZEROS: $s = 0$

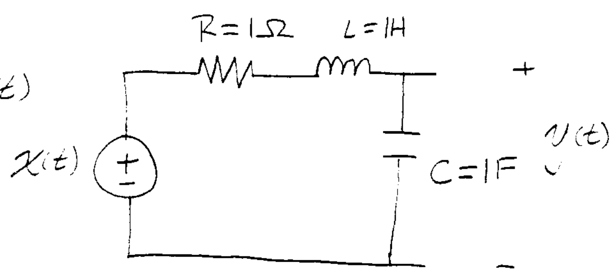
POLES: $s = -1$

$$= \frac{s^2}{(s+1)(s+1)}$$



$\boxed{\text{HIGHPASS}}$

18) a) $$0 = \frac{d^2y}{dt^2} + \frac{dy}{dt} + y(t) - x(t)$$



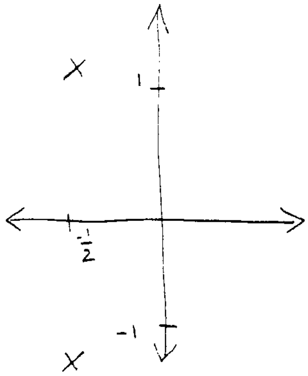
$$x(t) = \frac{d^2y}{dt^2} + \frac{dy}{dt} + y(t)$$

$$X(s) = s^2 Y(s) + sY(s) + Y(s)$$

$$X(s) = Y(s)(s^2 + s + 1)$$

$$\boxed{H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2+s+1} \quad \mathrm{Re}\{s\} > \frac{-1}{2}}$$

b) NO ZEROS, POLES: $s = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)} = \boxed{\frac{-1}{2} \pm \frac{i\sqrt{3}}{2}}$



c) $x(t) = \frac{d^2y}{dt} + 10^{-3}\frac{dy}{dt} + y(t)$

$X(s) = s^2 Y(s) + 10^{-3} s Y(s) + Y(s)$

$X(s) = Y(s)(s^2 + 10^{-3}s + 1)$

$$\boxed{H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2+10^{-3}s+1} \quad \mathrm{Re}\{s\} > -\frac{1}{2}10^{-3}}$$

d) $\boxed{\text{BANDPASS}}$

9.19) a) $x(t) = e^{-2t}u(t+1)$

$$X(s) = \int_0^\infty e^{-2t}u(t+1)e^{-st}\,dt = \int_0^\infty e^{(-2-s)t}\,dt = \frac{-1}{s+2}\left[e^{(-2-s)t}\right]_0^\infty$$

$$X(s) = \frac{-1}{s+2}(0-1) = \boxed{\frac{1}{s+2}}$$

b) $x(t) = \delta(t+1) + \delta(t) + e^{-2(t+3)}u(t+1)$

$$X(s) = \int_0^\infty (\delta(t+1) + \delta(t) + e^{-2(t+3)}u(t+1))e^{-st}\,dt$$

$$= \int_0^\infty \delta(t+1)e^{-st}\,dt + \int_0^\infty \delta(t)e^{-st}\,dt + \int_0^\infty e^{-2(t+3)}u(t+1)e^{-st}\,dt$$

$$= 0 + 1 + \int_0^\infty e^{t(-2-s)}e^{-6}\,dt$$

$$= 1 + \frac{-e^{-6}}{2+s}\left[e^{t(-2-s)}\right]_{0}^{\infty}$$

$$\boxed{X(s) = 1 + \frac{e^{-6}}{2+s}}$$

c) $x(t) = e^{-2t}u(t) + e^{-4t}u(t)$

$$X(s) = \int_0^{\infty} e^{-2t}u(t)e^{-st}dt + \int_0^{\infty} e^{-4t}u(t)e^{-st}dt$$

$$= \int_0^{\infty} e^{t(-2-s)}dt + \int_0^{\infty} e^{t(-4-s)}dt$$

$$= \frac{-1}{2+s}\left[e^{t(-2-s)}\right]_0^{\infty} - \frac{1}{4+s}\left[e^{t(-4-s)}\right]_0^{\infty}$$

$$= \frac{-1}{2+s}(0-1) - \frac{1}{4+s}(0-1) = \boxed{\frac{1}{2+s} + \frac{1}{4+s}}$$