Problems in Hellenistic and Hindu Mathematics

Wherever numbers are called for, use modern notation for numerals, fractions, and decimals.

1. Describe how Apollonius of Perga obtained the ellipse, parabola, and hyperbola by slicing a double-napped cone. What is a “subcontrary” circular section of a cone? Include pictures.

2. Describe (with a diagram) how Eratosthenes measured the circumference of the Earth.

3. What does the Sieve of Eratosthenes do and how does it work? Include a sketch.

4. State Ptolemy’s Theorem. Show how it can be used to derive trigonometric formulas.

5. What is meant by “precession of the equinoxes”? Who discovered it?

6. What do these words mean in Ptolemaic astronomy: equant, deferent, epicycle?

7. A triangle has sides with lengths 7, 8, and 12. What is its area?

8. An annulus, given in polar coordinates by \( \{(r, \theta) : 1 \leq r \leq 2\} \), is revolved about the axis \( x = 3 \). Find the volume of the resulting solid of revolution.

9. In class we used Pappus’ Theorem to find the center of gravity of the half-disk, \( \{(r, \theta) : 0 \leq r \leq 1, 0 \leq \theta \leq \pi\} \). Find the center of gravity of the quarter-disk, \( \{(r, \theta) : 0 \leq r \leq 1, 0 \leq \theta \leq \pi/2\} \). Use this result to find the center of gravity of the three-quarters disk, \( \{(r, \theta) : 0 \leq r \leq 1, 0 \leq \theta \leq 3\pi/2\} \). Hint: You can use Pappus’ Theorem here, but Archimedes-style balance arguments are probably easier.

10. Multiply 24 times 88, 59 times 127, and 115 times 739, using the “gelosia” method.

11. Do the divisions (quotient and remainder) 130 ÷ 16, 2258 ÷ 73, and 10894 ÷ 312, using the “galley” method.

12. What is the Hindu method of squaring a rectangle; that is, given a rectangle, constructing a square with the same area?

13. Suppose that there were 7 senses: the usual 5, plus sense of direction and sense of humor. If created things are classified according to the number and types of their senses (so that, for example, objects/creatures having the sense of smell and no others would constitute one class, objects/creatures having only the senses of smell and of humor would constitute another class, etc.), how many different classes would there be? Check your answer by computing, for each \( k \) (0 ≤ k ≤ 7), the number of classes of objects/creatures having exactly \( k \) senses, and adding up these numbers.