Newton's Law of Cooling

Introduction:

Have you ever thought about how long it takes for your cup of coffee or tea to cool down? Have you ever wondered what variables make an object cool faster or slower and how that rate is measured? These were the same types of questions that have lead numerous scientists to conduct studies on the nature of cooling since the beginning of the 18th century. The search to find a law that explained the nature of cooling was not easily established. The cooling process embeds the two contrasting phenomenons of convection and conduction, which makes it difficult to generate a law, especially because the two have very different properties. Additionally, the emission and absorption of thermal radiation and heat transfer are irregular because of the many variables that associate with them, making it even more challenging to develop a concrete law (Besson, 2010).

English physicist, Isaac Newton, conducted many experiments on the nature of cooling. Newton applied a principle to his experiments which states that the rate of heat loss will be proportional to the temperature difference of two bodies. This principle assisted him in estimating the rate at which a red-hot iron ball loses heat. He conducted this experiment by measuring the time it took the iron ball to go from an unknown red hot temperature to a known temperature. Then he compared that time with the amount of time it took for the iron ball to cool from the known temperature all the way through the range of lower and ordinary temperatures (Chisholm, 1910). In 1701, Newton published his findings about the relationship between the temperature of an object and the time it took to cool in an article in the Philosophical Transactions Of the Royal Society (Besson, 2010). In the article he wrote that,

> the heat which hot iron, in a determinate time, communicates to cold bodies near it, that is, the heat which the iron loses in a certain time is as the whole heat of the iron; and therefore (ideoque in Latin), if equal time of cooling be taken, the degrees of heat will be in geometrical proportion (Besson, p.4, 2010).

Newton analyzed his results from the experiment and generalized his conclusions to be a comprehensive hypothesis of the exponential law of cooling and not just an experimental result. Even though he didn't provide a written formula, he "noted that his law could be deduced mathematically from a linear relationship between temperature change rate and temperature difference between the object and the environment" (Besson, p.4, 2010). This discovery by Newton was significant because it was the first time that people were able to not only calculate how long it would take for an object to cool, but also it also measured temperatures at a newly discovered range of over 600 degrees celsius (Grigull, 1984).

Newton's discovery also resulted in him developing a thermometric scale. To develop his temperature scale, Newton used two types of temperature measuring tools. First, a linseed oil thermometer, to calculate the lower temperatures and second, a calorimeter to measure the high temperatures by calculating the rate of cooling temperature. He dedicated a significant part of his article to recording values of already known temperatures such as, the human body temperature, the boiling points of differing liquids, and for the first time freezing, melting, points of metals and alloys (Grigull, 1984). He used these known temperatures as reference points when measuring the temperature of the iron ball. From the point when the iron ball was initially taken out of the fire and the temperature was too high to be measured using a linseed oil thermometer, Newton calculated the temperature by measuring the time it took for the temperature to decrease and reach the same temperature as the human body. Newton made the conclusion that, "the excess of the degrees of the heat... were in geometrical progression when the times are in an arithmetical progression" (Besson, p.3, 2010).

What is Newton's Law of Cooling?

Newton's Law of Cooling states that the rate at which the temperature of a body loses heat is proportional to the difference in the temperature between a body itself and its surroundings ("Other Differential Equations", UBC Calculus). In mathematical terms $\frac{dT}{dt} = -k(T - T_a)$, where dT is the change in temperature of the object, dt is the change in time, k is a constant, and T_a is the ambient temperature. The positive constant, k, can differ depending on the other variables of the situation, such as the amount of surface area exposed to the original object or what the material is made out of. The equation $\frac{dT}{dt} = -k(T - T_a)$ is a separable differential equation. To solve this equation for a general solution, one would begin by manipulating the equation so that all the time variables are on one side and all of the temperature variables are on the other side. First divide both sides by $(T - T_a)$ and multiply both sides by dt to get $\frac{1}{T-T_a} * (dT) = -k * dt$. After this step is done, one must integrate both sides of the equation, $\int \frac{1}{T - T_a} * dT$ and $\int - k * dt$. The solution to the equation when you integrate it is $ln|T - T_a| = kt + C$ when C is a new constant. To isolate the temperature on one side, it results in $|T - T_a| = e^{-kt}e^c$. If the desire is to find the temperature of an object which was originally equal or hotter than the ambient temperature, then the function would look like, $T(t) = e^{-kt}e^c + T_a$ and if the desire was to find the temperature of an object that was originally cooler than the ambient temperature and was warming then, $T(t) = T_a - e^{-kt}e^c$. These are the two general solutions to the original differential equation using the logic of Newton's Law of Cooling ("Newton's Law of Cooling", 2016).

The Application of Newton's Law of Cooling

The formulas derived above from Newton's Law of Cooling can easily be applied to real life applications. If we are looking at a scenario where the temperature of an object is greater than the temperature of the ambient temperature, such as a bowl of hot soup, then we would use the formula $T(t) = Ce^{-kt} + T_a$ to solve for the amount of time it will take the soup to cool to a specific temperature. To make the equation more manageable, we replaced e^c with the variable *C*. Now we can easily apply this equation to a real life scenario. For example, say we know that the ambient temperature in the room is 20 degrees celsius, the soup is originally 80 degrees celsius, and, from previous experiments, we know that after two minutes the soup will be 60 degrees celsius. Written in mathematical notation we have, $T_a = 20℃$, $T(0) = 80℃$, and $T(2) = 60℃$. If the goal is to see how long it will take for the bowl of soup to reach 40 ℃, then we must begin by solving the equation by finding the values to the two constants in the equation, *k* and *C*. For every scenario, the values of the constants will be different because of the different variables that occur in each setting. In this experiment we will use our known variables to help us solve for the unknown values of the constants.

First, we will start by plugging in the already known values into the original equation. If we start by using the $T(0) = 80℃$ and $T_a = 20℃$, then we can state that $T(0) = 80 = Ce^{-k*0} + 20$. Since we know that when *e* is raised to zero, it is equal to one the equation then becomes, $80 = C + 20$ and we can then solve for *C* to find that it is equal to 60 ℃.

Now, we must solve for the constant *k*. To do that we will use the information that $T(2) = 60℃$ and apply it to our new equation using the *C* value we found, $T(t) = 60e^{-kt} + 20$. So, our new equation would be, $T(2) = 60 = 60e^{-2k} + 20$. To solve for *k*, we would first add negative 20 to both sides and then divide both sides by 60 to get $\frac{2}{3} = e^{-2k}$. Next, take the natural log of both sides, $ln\,\frac{2}{3} =- 2k$ and from there we know that $k =- \frac{1}{2}\,ln\frac{2}{3}$.

Now we can rewrite our main equation using the two newly found values of the constants *C* and *k*, $T(t) = 60e^{(\frac{1}{2}ln\frac{2}{3})t} + 20$. If we look back at the original goal of finding out how long it would take for a bowl of soup to cool down to 40 ℃, we would then write the equation as $40 = 60e^{(\frac{1}{2}ln\frac{2}{3})t} + 20$. To solve for the time, *t*, we first would add negative 20 to both sides and then divide both sides by 60 to get, $\frac{1}{3} = e^{(\frac{1}{2}ln\frac{2}{3})t}$. Next we would take the natural log of both sides and then divide both sides by $(\frac{1}{2}ln\frac{2}{3})$ to isolate the variable *t* on one side, $t = \frac{2ln(\frac{2}{3})}{ln(\frac{2}{3})}$. When you round this out to the nearest hundredth it comes out to 5.42, which means that it would take 5.42 minutes for the bowl of soup to cool to 40℃ ("Applying Newton's Law", 2016). This is just one example of how Newton's Law of Cooling can help to calculate real world problems.

Conclusion:

To reiterate, Newton's Law of Cooling states that the rate of change of an object's temperature is proportional to the difference in temperature between that object and the ambient, surrounding temperature. This denotes that the greater the difference in temperature between the object and its surroundings, the faster the object will cool. On the contrary, when the object's temperature and the ambient temperature are closer, then the object will cool down more slowly. However, while some scientists confirmed Newton's law, such as G. W. Richmann and Johann Heinrich Lambert, other scientists found discrepancies in Newton's discovery (Besson, 2010). George Martine pointed out that Newton's law was more mathematical than physical and that it doesn't represent a true view of nature. Martine provided the example that, "the heat of a body does not really decrease in that exact proportion" (Besson, p.6, 2010). Inaccuracies within Newton's law were also discovered when the temperatures being recorded were relatively high, creating large percentages of error in the calculations.

In 1817, over a century after Newton originally published his work, physical chemist Pierre Dulong and physicist Alexis Petit corrected Newton's original Law of

Cooling and formulated that, “the quickness of cooling for a constant excess of temperature, increases in geometrical progression, when the temperature of the surrounding space increases in the medical progression; whereas, according to Newton's law, this quickness would not have very that all” (Whewell, 1866). In general, it seemed that the scientists who were more empiricist were the ones to discredit the validity of Newton’s work. Alternatively, the scientists who were more theoretical had a tendency to try to preserve Newton’s original law and had confidence in the simplicity of the natural laws. Even though Newton’s Law of Cooling had its discrepancies, it was still foundational in the discussion about the concept of a proportionality between cause and effect, and it influenced the way many future scientists analyzed the nature of cooling.

Works Cited

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