1. Use the method of power series to find the recurrence relation for \( y' - xy' - 4y = 0 \).

\[
y = \sum_{k=0}^{\infty} a_k x^k = \sum_{k=0}^{\infty} k(k-1) a_k x^{k-2} - \sum_{k=0}^{\infty} k a_k x^k - 4 \sum_{k=0}^{\infty} a_k x^k = 0
\]

\[
= \sum_{k=0}^{\infty} \left( \frac{(k+1)(k+2)}{2} a_{k+2} x^k ight)
\]

Thus,

\[
\sum_{k=0}^{\infty} \left( \frac{(k+1)(k+2)}{2} a_{k+2} - (k+4) a_k \right) x^k = 0
\]

\[
\Rightarrow a_{k+2} = \frac{k+4}{(k+2)(k+1)} a_k,
\]

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\[
2. The use of power series on a differential equation leads to the recurrence relation \( a_{k+2} = \frac{k-5}{k+3} a_k \) for \( k = 0, 1, 2, \ldots \). Find the general solution to the differential equation.

\[
a_2 = -\frac{5}{3} a_0,
\]

\[
a_3 = -\frac{4}{4} a_1 = -a_1,
\]

\[
a_4 = -\frac{3}{5} a_2 = a_0,
\]

\[
a_5 = -\frac{2}{6} a_3 = \frac{1}{3} a_1,
\]

\[
a_6 = -\frac{1}{7} a_4 = -\frac{1}{7} a_0,
\]

\[
a_7 = \frac{6}{8} a_5 = 0,
\]

\[
a_{2m+1} = a_0, \; m = 3, 4, \ldots
\]

\[
a_{2m} = \frac{(-5)(-7)(-9) \cdots (2m-7)}{3 \cdot 5 \cdot 7 \cdots (2m+3)} a_0,
\]

\[
2m = 1, 2, \ldots
\]

\[
y = a_0 + a_0 \sum_{m=1}^{\infty} \frac{(-5)(-7)(-9) \cdots (2m-7)}{3 \cdot 5 \cdot 7 \cdots (2m+3)} x^{2m} + a_1 \left( x - x^3 + \frac{1}{3} x^5 \right)
\]