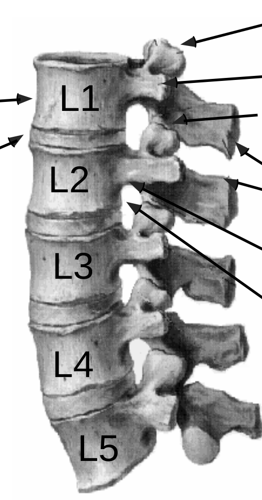
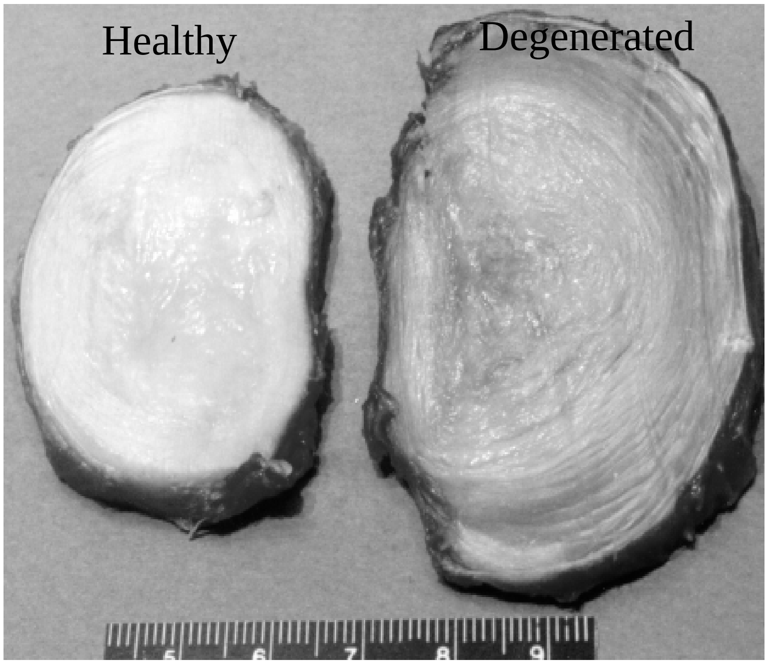
# Biphasic viscoelasticity introduction

James Iatridis, PhD
School of Engineering
UVM

# Lumbar Spine




Netter, 1987




# Multiphasic modeling in the intervertebral disc

Motivations for applying multiphasic modeling of the disc

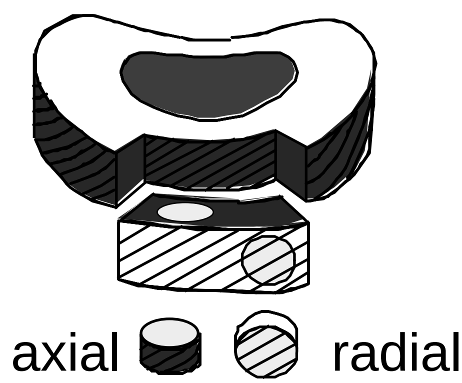
- Swelling
- Transport in the disc
  - Large avascular structure
  - Very long diffusion distances
  - Significant transport between waking (upright) and resting (reclined)
  - Calcification of endplates with aging and degeneration
- Electical phenomena
- pH effects

## Compressive Properties of Anulus Fibrosus

Iatridis et al., 1998

- Confined compression of cylindrical specimens
- Finite deformation biphasic model
  - Compressive modulus: $H_{A0}$, β
  - Strain-dependent permeability: $k_0$, M
- Effects of orientation
- Effects of large deformations
- Effects of degeneration

## Specimen Harvest



## Biphasic Theory

- finite deformation formulation
  - Mow et al, 1980
- continuity equation

  $\nabla \bullet (\phi^s \mathbf{v}^s + \phi^f \mathbf{v}^f) = 0$
- momentum equations

  $\nabla \bullet \boldsymbol{\sigma}^s + \boldsymbol{\pi} = 0 \quad \nabla \bullet \boldsymbol{\sigma}^f - \boldsymbol{\pi} = 0$
- momentum transfer

  $\pi = (\phi^f)^2 (v^f - v^s)/k + p \nabla \phi^s$
- constitutive equations

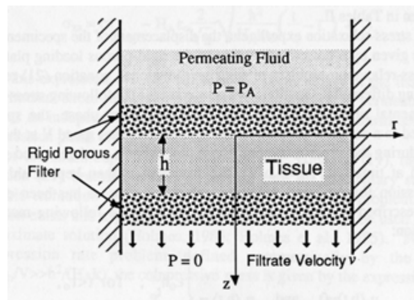
  $\boldsymbol{\sigma}^s = -\phi^s p \mathbf{I} + \boldsymbol{\sigma}^e \qquad \boldsymbol{\sigma}^f = -\phi^f p \mathbf{I}$

## Constitutive relationships

- These constitutive relationships are required for the solid and fluid phases, and an additional relationship is required for the permeability.
- In general, the fluid viscosity is assumed zero (inviscid), but this is not necessary in the general formulation of Mow et al., 1980.
- In this talk, we will motivate different relationships for solid phase and permeability with experimental data.

## 1-D permeation test -- convection

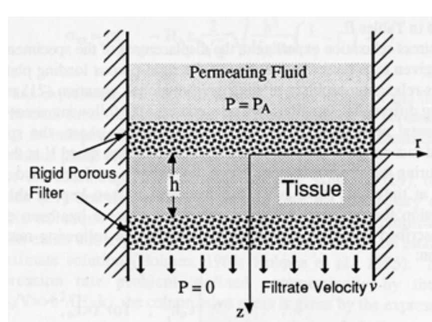
- Pressure gradient causes flow of fluid through the tissue.
- Permeability of tissue to water flow can be calculated from pressure gradient, area, flow rate
- Note that boundary conditions are necessary for both solid and fluid phases of tissue.
- Navier's equations of motion simplify to Darcy's law when considering boundary conditions and steady state behaviors.




## Darcy's Law

- $k=Qh/(AP_A)$

  Where,
  - k=apparent permeability
  - Q=volumetric flow rate
  - A=cross-sectional area
  - $P_A$=applied pressure




Also see:
http://en.wikipedia.org/wiki/Darcy's_law

## Strain-dependent Permeability

- Permeability is a function of strain in the tissue
- Strain in the tissue is affected by
  - applied strain -- grip to grip strain
  - Applied pressure -- drag-induced strain

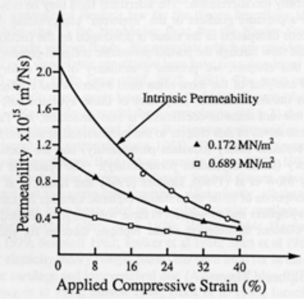



Figure 8. Permeability of articular cartilage, at two levels of pressure $P_A$, as a function of the applied clamping strain. The intrinsic permeability is obtained from the limited case of $P_A \rightarrow 0$ from six levels of $P_A$.

**Strain-dependent permeability**

$k = k_0 \exp(M\varepsilon)$

where,

$k_0$=zero-strain (intrinsic) perm

M=strain-dep perm coeff

$\varepsilon$=dilatational strain

## Finite deformation permeability relationship

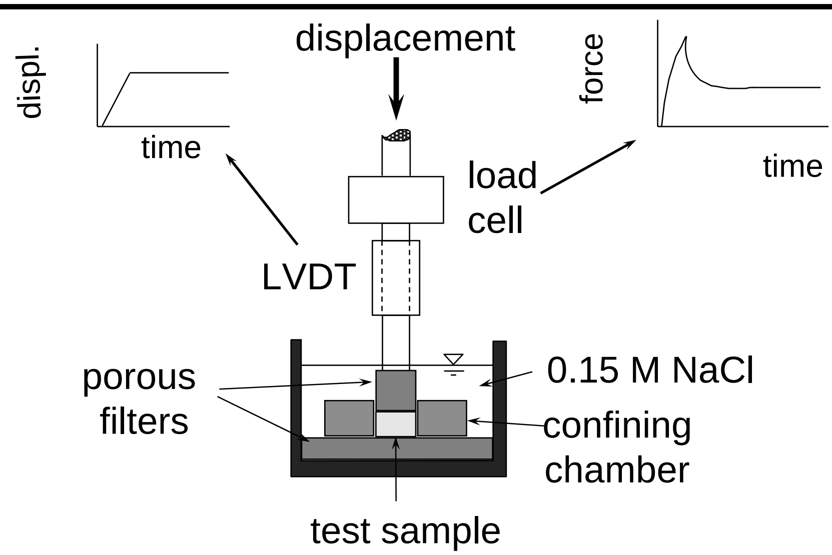
- Finite deformation formulation
- strain-dependent permeability ($k_o$, M)

Holmes & Mow, 1990, Warden et al, 1994

# Confined compression

- Modulus (directly) and permeability (indirectly) can also be evaluated from compression tests.
- Confined compression offers a relatively simple analytical solution. It works quite well for intervertebral disc tissue, but not as well for hyaline cartilage.

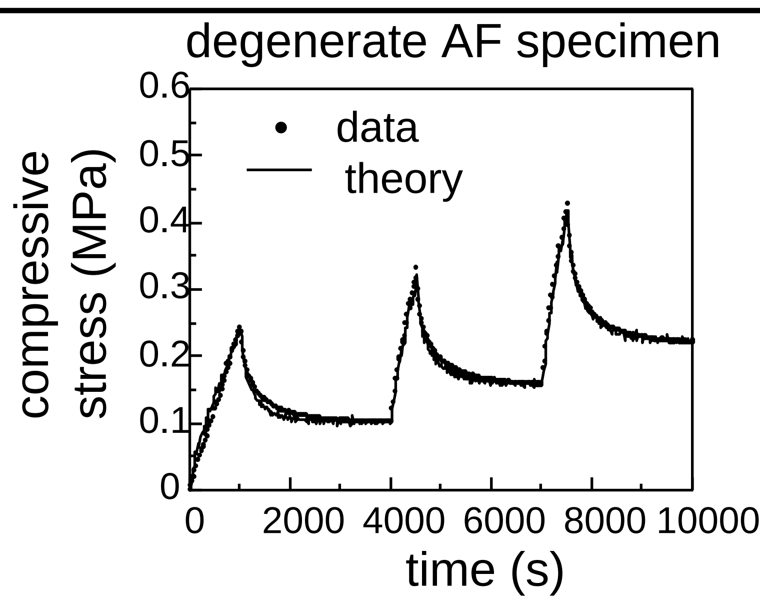
# 1-D Confined Compression: Can be creep or stress relaxation test



# Compression

- Viscoelastic contributions are largely from water squeezing out of the tissue
  - biphasic or poroelastic viscoelasticity

# Stress Relaxation



# Biphasic Theory

- finite deformation formulation
  - Mow et al, 1980
- continuity equation

  $\nabla \bullet (\phi^s \mathbf{v}^s + \phi^f \mathbf{v}^f) = 0$
- momentum equations

  $\nabla \bullet \boldsymbol{\sigma}^s + \boldsymbol{\pi} = 0 \quad \nabla \bullet \boldsymbol{\sigma}^f - \boldsymbol{\pi} = 0$
- momentum transfer

  $\pi = (\phi^f)^2(v^f - v^s)/k + p\nabla\phi^s$
- constitutive equations

  $\boldsymbol{\sigma}^s = -\phi^s p\mathbf{I} + \boldsymbol{\sigma}^e \qquad \boldsymbol{\sigma}^f = -\phi^f p\mathbf{I}$

# Biphasic Theory

- Elastic relationship can be linear (small strain) or nonlinear (finite deformation).
  - Linear theory ($H_A$) $\sigma^e = H_A \varepsilon$
  - Nonlinear elastic stress-stretch ($H_{Ao}$, $\beta$)

Holmes & Mow, 1990, Warden et al, 1994

# Linear solution to biphasic creep

- Navier's equations of motion reduce to the diffusion equation

$$H_A k \frac{\partial^2 u_z}{\partial z^2} = \frac{\partial u_z}{\partial t}$$

**Creep Problem**

- Boundary conditions:
  - uz(h,t)=0
  - duz/dz=-F0U(t)/HA, U(t) = unit step function

Solution

$$\frac{u_z(0,t)}{h} = \frac{F_0}{H_A}\left[1 - \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(n+0.5)^2}\exp\left(-\frac{H_A k(n=0.5)^2\pi^2}{h^2}t\right)\right]$$

# Nonlinear solution to biphasic stress relaxation

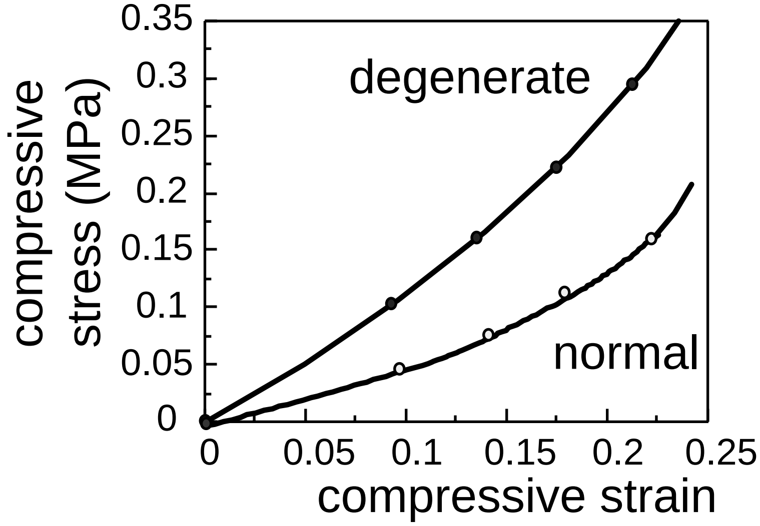
- nonlinear elastic stress-stretch ($H_{Ao}$, $\beta$)

- strain-dependent permeability ($k_o$, M)

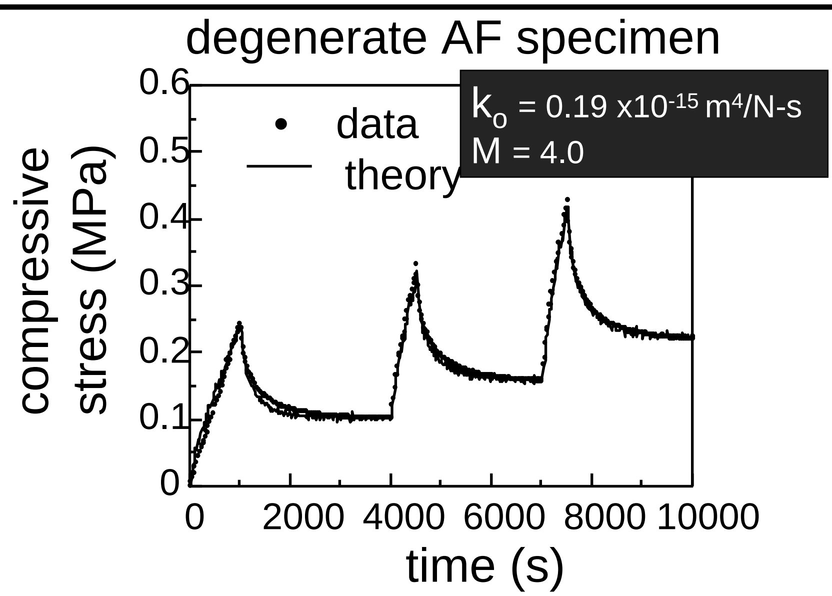
- A computational solution is used w/ 2 part curve-fit

Holmes & Mow, 1990, Ateshian et al, 1996

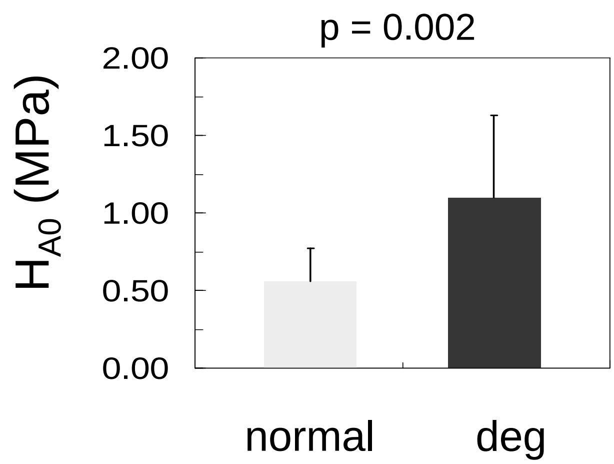
# Equilibrium Stress - Strain



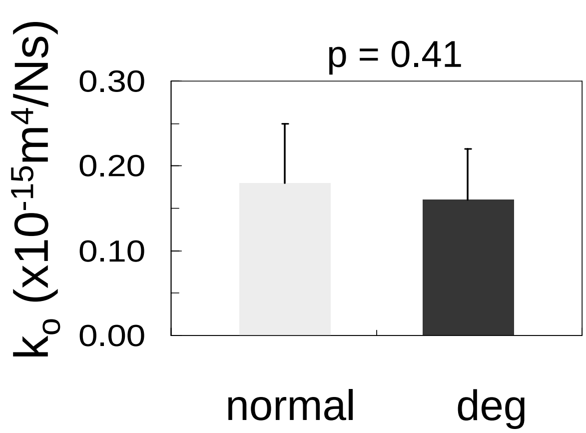

# Stress Relaxation



# Aggregate Modulus




# Zero-Strain Permeability



## Summary

- Significant alterations in mechanical behaviors of intervertebral disc tissue occur from degeneration
- Mechanical properties may be a sensitive index of pathological changes
- Viscoelasticity occurs by 2 mechanisms
  - flow-dependent (biphasic) under volume changes
  - flow-independent (intrinsic) in the absence of volume changes
- Distinguishing between these 2 mechanisms is important when considering tissue remodeling and mechano-biology

## Future directions

- Specific biphasic problems and solutions
- Comparison with experimental data