12. \( x(t) = \delta(t-1) - \delta(t+1) \)

(a) From Table E.2 p.949

\( x(t) \leftrightarrow 1 \)

Expanding p.333

\[ x(t) \leftrightarrow e^{-j\pi t} x(jw) \]

\[ x(t) = \delta(t-1) - \delta(t+1) \rightarrow (1)e^{-j\pi w} - (1)e^{j\pi w} = -2j \sin w \]

(b) Now, another way.

Consider

\[ y(t) \]

From Table E.2 p.949 we have

\[ x(t) \leftrightarrow \text{Sinc}(w/2\pi) \]

\[ \frac{w}{2} \equiv \frac{\sin(w/2)}{w/2} \]

that

we see that \( y(t) = -w/2 \) \( \implies y(jw) = -\frac{w}{2j} \) \( \text{(Using scaling property)} \)

\[ = -2 \frac{\sin(w)}{w} \]

we also see that \( x(t) = \frac{d}{dt} y(t) \)

\[ \implies x(jw) = jw y(jw) = jw \left( -2 \frac{\sin(w)}{w} \right) = -2j \sin w \]

which confirms with direct result.
\[ x(t) = \text{rect}(t) \cdot \frac{1}{2} \text{comb}(t) \]

\[
\begin{align*}
X(\omega) &= \frac{1}{2} \int_{-1/2}^{1/2} e^{-j\omega t} \text{rect}(t) \, dt \\
&= \frac{1}{2} \int_{-1/2}^{1/2} \left[ e^{j\omega t} - e^{-j\omega t} \right] \\
&= \frac{1}{2} \left[ \frac{1}{j\omega} \left( e^{j\omega/2} - e^{-j\omega/2} \right) \right] = \frac{1}{2} \frac{\sin(\omega/2)}{\omega}
\end{align*}
\]

\[
X(j\omega) = \int_{-1/2}^{1/2} e^{-j\omega t} \, dt = \frac{1}{2} \left[ e^{-j\omega t} - e^{j\omega t} \right]_{-1/2}^{1/2} = \frac{1}{2} \frac{\sin(\omega/2)}{j\omega}
\]

Consequently, \( T \cdot X(\omega) = 2 \cdot X(\omega) \) with \( X(j\omega) \)

\[ x(t) \bigg|_{\omega=\omega_0} = 2 \cdot x(t) \] as should be.
4 (a) \[ x(t) = 4 \text{sinc} \left( \frac{t}{5} \right) = 4 \frac{\sin \left( \frac{\pi t}{5} \right)}{\pi t/5} \]

Parseval's Theorem
\[ \int |x(t)|^2 dt = \frac{1}{2\pi} \int |X(j\omega)|^2 d\omega \]

From E.T. sours p. 947, we know
\[ \text{sinc}(t) \leftrightarrow \text{rect} \left( \frac{w}{2\pi} \right) \]
\[ \therefore 4 \text{sinc}(t) \leftrightarrow 4 \text{rect} \left( \frac{w}{2\pi} \right) \]

and \[ 4 \text{sinc} \left( \frac{\pi t}{5} \right) \leftrightarrow \frac{4}{|1/5|} \text{rect} \left( \frac{w}{2\pi} \frac{5}{\pi} \right) \]

\[ a = \frac{\pi}{5} \]

\[ +\frac{\pi}{5} \]

\[ \frac{1}{2\pi} \int |\frac{2}{5}|^2 d\omega = \frac{1}{2\pi} \left( \frac{2}{5} \right)^2 \]

\[ = \frac{2\pi}{5} \]

\[ \frac{2\pi}{5} = \frac{160}{160} \]

17. Sketch my a phase of FT of the signals:

(a) \[ x(t) = \delta(t - 2) \]

\[ x(j\omega) = 1 \quad e^{-jw^2} \]

\[ |x(j\omega)| = 1 \quad \text{all } \omega \]

\[ \theta(j\omega) = -2\omega \quad \text{all } \omega \]
(b) \( x(t) = u(t) - u(t-1) \)

\[ x(jw) = \frac{1}{jw} \left[ \frac{1}{jw} + \pi \delta(w) \right] \]

\[ = \frac{1}{jw} \left( 1 - e^{-jw} \right) + \pi \delta(w) \left( 1 - e^{-jw} \right)^{-1} = 0 \]

\[ = \frac{e^{-jw/2}}{jw} \left( e^{jw/2} - e^{-jw/2} \right) + \pi \delta(w) e^{-jw/2} \left( e^{jw/2} - e^{-jw/2} \right) \]

\[ = \frac{e^{-jw/2}}{jw} 2 \sin(w/2) + \pi \delta(w) e^{-jw/2} 2j \sin(w/2) \]

\[ x(jw) = e^{-jw/2} \frac{\sin(w/2)}{jw/2} \]

\[ |x(jw)| = \left| \frac{\sin(w/2)}{jw/2} \right| \theta(w) = -\frac{w/2}{\pi} + \pi \text{ whenever } \frac{\sin(w/2)}{w/2} \text{ changes sign.} \]
\[
\begin{align*}
   e^{x(t)} &= 6 \delta_m(2\pi n t) = 6 e^{j 2\pi n t} - 6 e^{-j 2\pi n t} \\
   i &\to 2\pi \delta(w) \quad (p. 949 \ T_8x) \\
   e^{j w_0 t} &\to 2\pi \delta(w-w_0) \\
   K e^{j w_0 t} &\to K(2\pi \delta(w-w_0)) \\
   \therefore x(t) &\to 6 \frac{2\pi}{2j} \delta(w-2\pi n) - 6 \frac{2\pi}{2j} \delta(w+2\pi n) \\
   &= -6j\pi \delta(w-2\pi n) + 6j\pi \delta(w+2\pi n) \\
   \end{align*}
\]
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(c) \( X(j\omega) = \frac{j\pi \delta(\omega + \omega_0)}{(\omega + j\omega_0)} - \frac{j\pi \delta(\omega - \omega_0)}{(\omega - j\omega_0)} \)

We know \( x(t) = \sin(10\pi t) = \frac{1}{2j} e^{j10\pi t} - \frac{1}{2j} e^{-j10\pi t} \)

\( \rightarrow \frac{1}{2j} 2\pi \delta(\omega - 10\pi) - \frac{2}{2j} 2\pi \delta(\omega + 10\pi) \)

\( = -j\pi \delta(\omega - 10\pi) + j\pi \delta(\omega + 10\pi) \)

\[ \therefore \text{given } X(j\omega) \rightarrow \sin(10\pi t) \]

(e) \( Y(j\omega) = \frac{5\pi}{j\omega} + 10\pi \delta(j\omega) \)

We know \( u(t) \rightarrow \frac{1}{j\omega} \delta(j\omega) \)

\( \therefore \sinu(t) \rightarrow \frac{\pi}{j\omega} + 5\pi \delta(j\omega) \)

(f) \( X(j\omega) = \frac{6}{3 + j\omega} \)

We know \( e^{-at}u(t) \rightarrow \frac{1}{j\omega + a} \quad a > 0 \)

\[ \therefore 6e^{-3t}u(t) \rightarrow \frac{6}{3 + j\omega} \]

\[ \therefore \frac{6}{3 + j\omega} \rightarrow 6e^{-3t}u(t) = x(t) \]