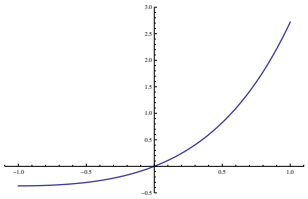
NAME:

Mid Term Exam I
Math 22—Spring 2009, Dummit

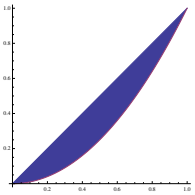
DIRECTIONS: This is a 1-hour exam. There are a total of 8 problems on this exam, *in no particular order*, with points as indicated, for a total of 100 points. There is no penalty for guessing on the multiple choice questions. Indicate your answers CLEARLY and NEATLY for partial credit. NO CALCULATORS are allowed.

1. (10) Find the average value of the function $f(x) = xe^x$ on the interval $[-1, 1]$. The graph of this function is shown.



2. (10) Sketch the region enclosed by $y = \cos x$ and $y = \sin x$ from $x = 0$ to $x = \pi/4$ and find the area of the region.

3. (25) The region bounded by the curves $y = x^2$ and $y = x$ (as shown) is rotated about the y-axis to form a solid D.
   (a) Set up an integral for the volume of D using the "washer method".
   (b) Set up an integral for the volume of D using "cylindrical shells".
   (c) Use either of the two integrals in (a) or (b) to compute the volume of D.



4. (10) Which of the following is the best cubic polynomial approximation for $f(x) = \sin 2x$ near $x = 0$?

(a) $1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$ (b) $1 - 2x^2$ (c) $x - \frac{1}{6}x^3$ (d) $2x - \frac{2}{3}x^3$ (e) $2x - \frac{4}{3}x^3$

5. (10) An aquarium 2 meters long, 1 meter wide and 1 meter deep is full of water. Find the work needed to pump half of the water out of the aquarium. (Use the fact that the density of water is 1000 kg/m^3).

6. (25) Evaluate the following integrals:

a. $\int x^2 \ln x \, dx =$

b. $\int \frac{\ln x}{x} \, dx =$

c. $\int t \sin 2t \, dt =$

d. $\int \sin^3 2x \cos^3 2x \, dx =$

e. $\int \arcsin x \, dx =$

f. $\int_{-2\pi}^{2\pi} \sin^2 x \, dx =$

7. (5) To evaluate the integral $\int_0^1 \dfrac{x^3}{\sqrt{5-2x^2}} \, dx$, one might use the substitution:

(a) $x = \sqrt{5} \sin\theta$ (b) $x = (5/2) \sin\theta$ (c) $x = \dfrac{\sqrt{10}}{2} \sin\theta$

(d) $x = \sqrt{5} \tan\theta$ (e) $x = \sqrt{5/2} \sec\theta$

8. (5) The substitution $x = 3\tan\theta$ transforms the integral $\int_0^3 x\sqrt{x^2+9} \, dx$ to the integral:

(a) $27 \int_0^{\pi/2} \sec^3\theta \tan\theta \, d\theta$ (b) $27 \int_0^{\pi/4} \sec^3\theta \tan\theta \, d\theta$ (c) $9 \int_0^{\pi/4} \sec\theta \tan\theta \, d\theta$

(d) $81 \int_0^{\pi/4} \sec^2\theta \tan^2\theta \, d\theta$ (e) $81 \int_0^{\pi/4} \sec^4\theta \tan\theta \, d\theta$